Crustal data

1. Topography
   – MOLA
2. Gravity
   – Theory (spherical harmonics, geoid, etc.)
   – Observations
   – Crustal Thickness

- Seismic (only on the Moon)

3. How does the crust respond to surface loads?
   – Lithospheric Flexure
   – Rheology of the lithosphere
   – Heat Flow
   – Admittance studies

Spherical Harmonics

Spherical harmonics in spherical coordinates are analogous to sines and cosines in Cartesian coordinates.

– Any function in spherical coordinates can be expressed as a sum of spherical harmonics

$$T(\theta,\phi) = \sum_{i=1}^{2}\sum_{l=0}^{l_{max}}\sum_{m=0}^{l} T_{ilm} Y_{ilm}(\theta,\phi) \qquad T_{ilm} = \frac{1}{4\pi}\int_S T(\theta,\phi)\, Y_{ilm}(\theta,\phi) \sin\theta\, \partial\theta\, \partial\phi$$

where T_{ilm} is a spherical harmonic coefficient, Y_{ilm} is a spherical harmonic function,

$$Y_{1lm} = P_l^m(\cos\theta)\cos m\phi$$

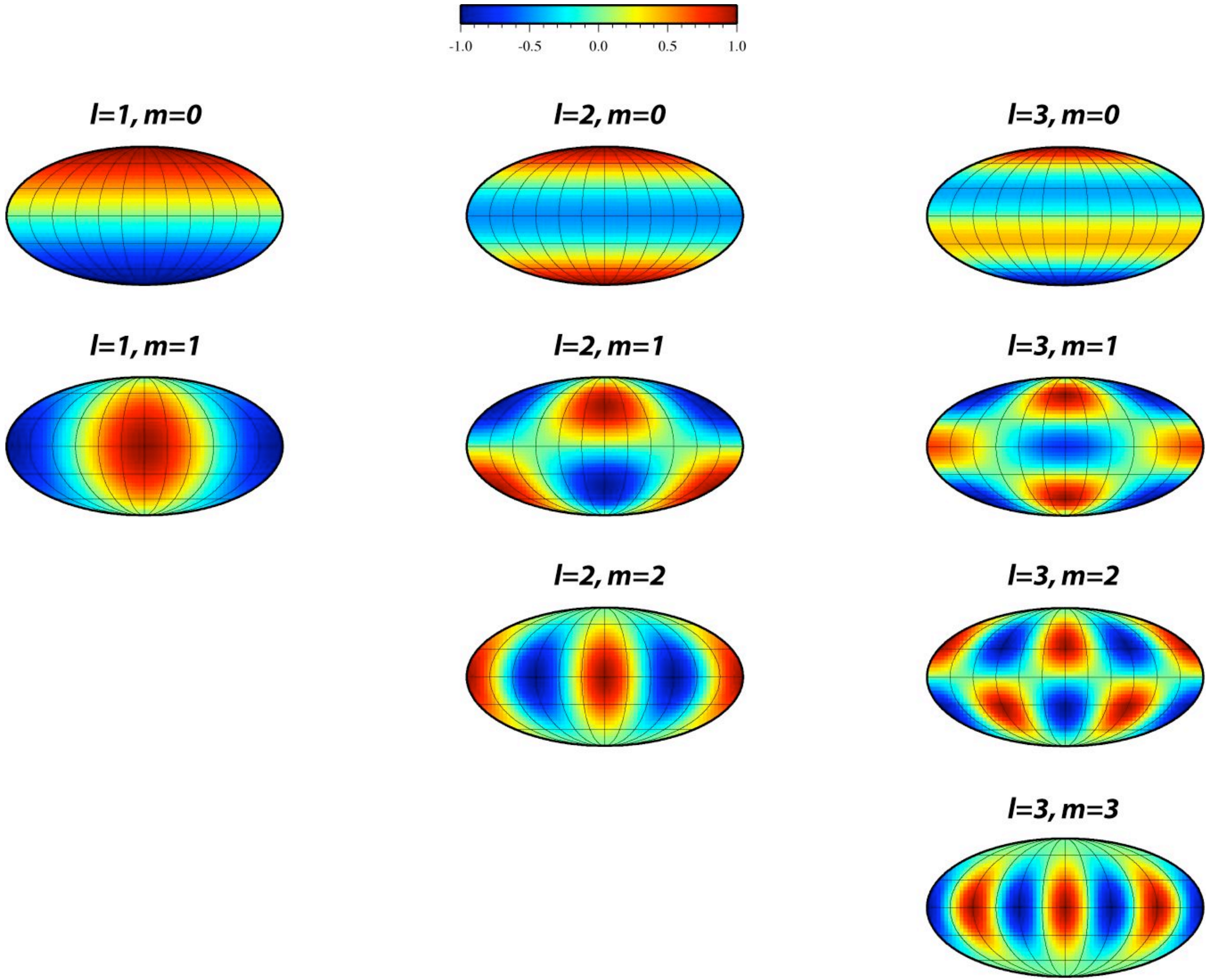
$$Y_{2lm} = P_l^m(\cos\theta)\sin m\phi$$

and the P_{lm}s are associated Legendre Polynomials,

– The spherical harmonics have $2m$ zeros in longitude, and l-m zeros in latitude

– Be careful with the normalization used! Geophysicists use

$$\int_\Omega Y_{ilm}\ Y_{i'l'm'}\, d\Omega = 4\pi\ \delta_{ii'}\ \delta_{ll'}\delta_{mm'}$$



-1.0
-0.5
0.0
0.5
1.0
l=1, m=0
l=2, m=0
l=3, m=0
l=1, m=1
l=2, m=1
l=3, m=1
l=2, m=2
l=3, m=2
l=3, m=3

Gravity

It is usually easier to work with scalar potentials, as opposed to a vectorial (three component) gravity field

$$\vec{g}(\theta,\phi) = \nabla U(\theta,\phi)$$

$$U(r,\theta,\phi) = \int_V \frac{G\,\rho(r',\theta,\phi)}{|r - r'|}\partial V$$

Since the gravitational potential satisfies Laplace's equation exterior to a mass distribution

$$\nabla^2 U = 0$$

the potential and radial gravity anomaly exterior to a planet can be expressed in spherical harmonics as

$$U(r,\theta,\phi) = \frac{GM}{r}\sum_{i=1}^{2}\sum_{l=0}^{l_{\max}}\sum_{m=0}^{l}\left(\frac{R}{r}\right)^{l} C_{\mathrm{ilm}} Y_{\mathrm{ilm}}(\theta,\phi)$$

$$g(r,\theta,\phi) = -\frac{GM}{r^2}\sum_{i=1}^{2}\sum_{l=0}^{l_{\max}}\sum_{m=0}^{l}(l+1)\left(\frac{R}{r}\right)^{l} C_{\mathrm{ilm}} Y_{\mathrm{ilm}}(\theta,\phi)$$

where the potential coefficients are related to the mass distribution of the planet by

$$C_{ilm} = \frac{1}{(2l+1)MR^l}\int_V r^l\,\rho(r,\theta,\phi)\,Y_{ilm}(\theta,\phi)\,dV$$

The Geoid

The geoid is the elevation (h_g) above the surface to an equipotential surface.

Since

$$\vec{g}(\theta,\phi) = \nabla U(\theta,\phi)$$

when one is on the geoid (or any equipotential surface), there are no lateral forces. Since oceans have no shear strength, their elevation will conform to an equipotential surface.

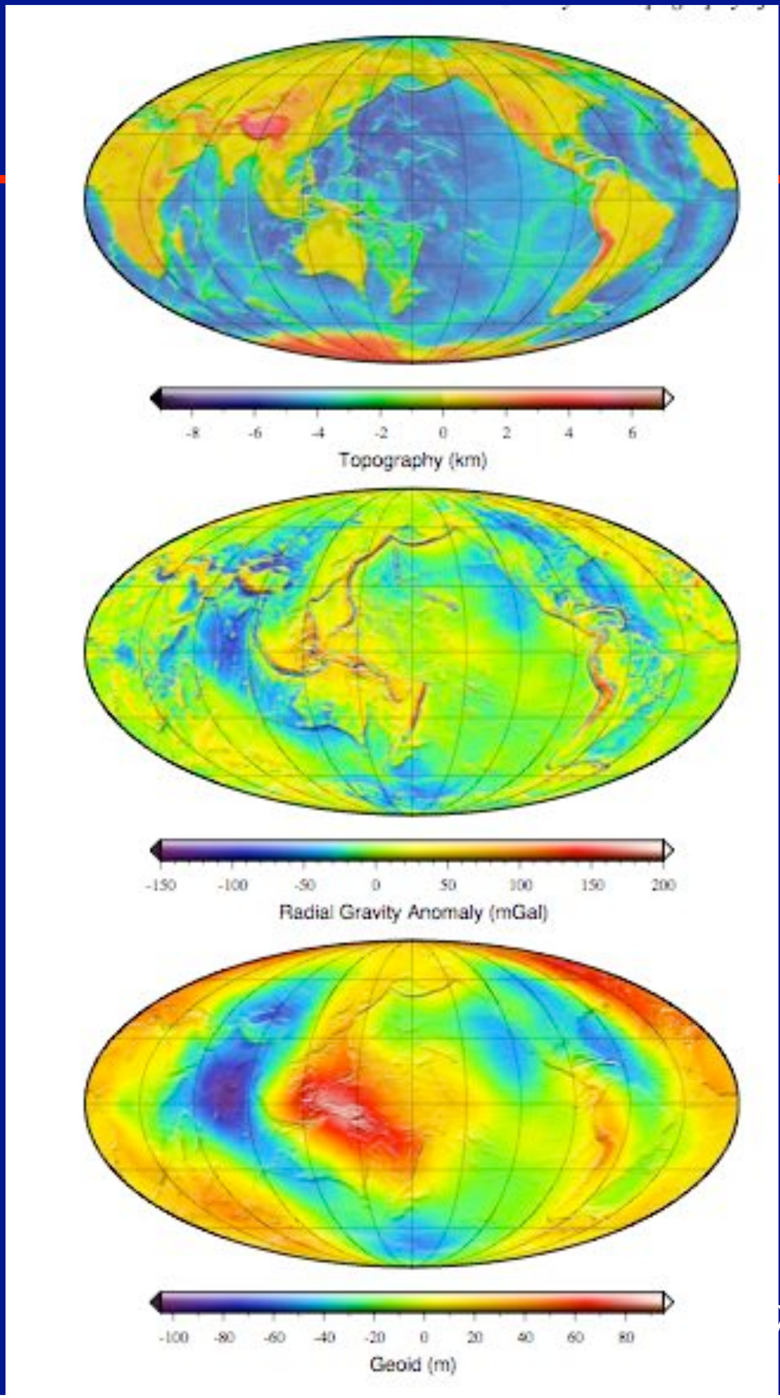
<u>How to Calculate the Geoid (to first order):</u>

$$U(R+h,\theta,\phi) \approx U(R,\theta,\phi) + \frac{dU}{dr}h = U(R,\theta,\phi) - g\,h$$

Letting $U(\mathrm{r+h},\theta,\phi)$ = constant

$$h_g = \frac{U(R,\theta,\phi)}{g} + \text{Constant} = R\sum_{l}\sum_{l}\sum_{m} C_{lm} Y_{lm}(\theta,\phi)$$

Geoids…



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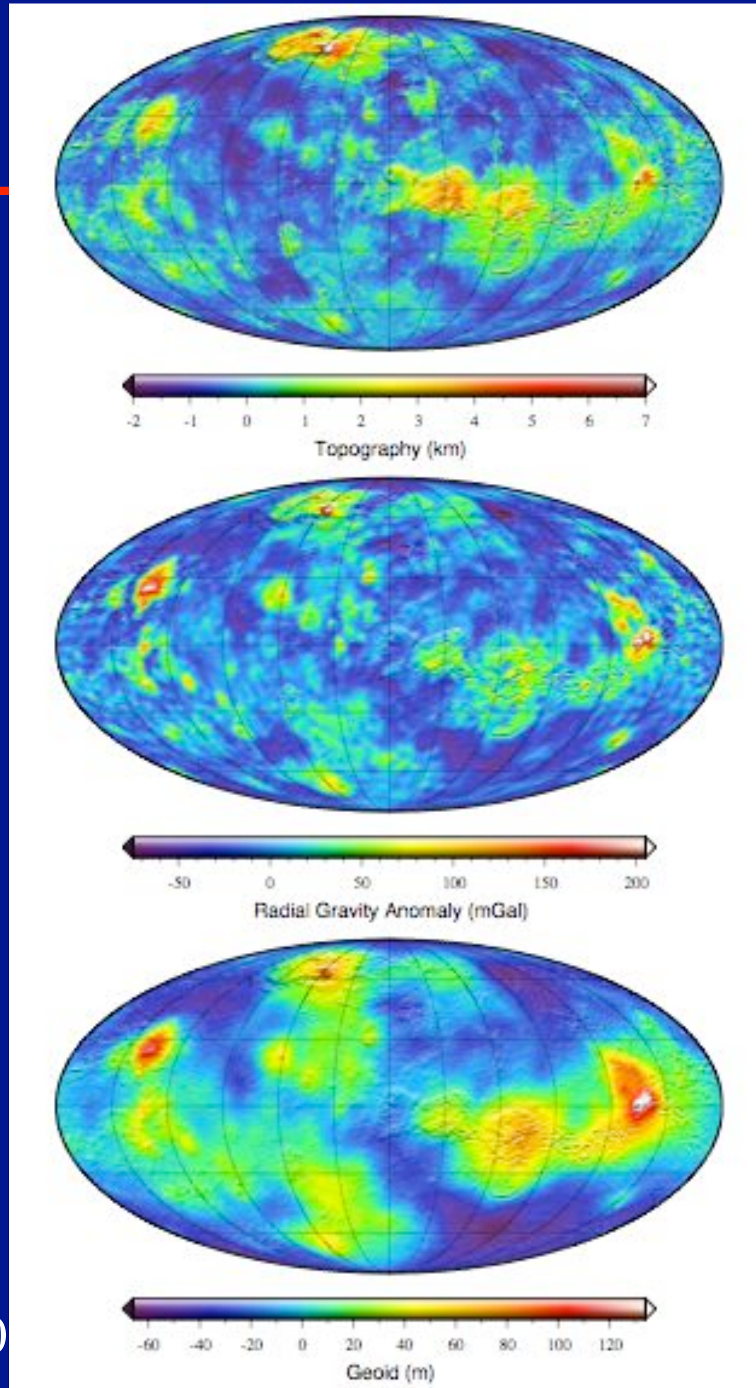
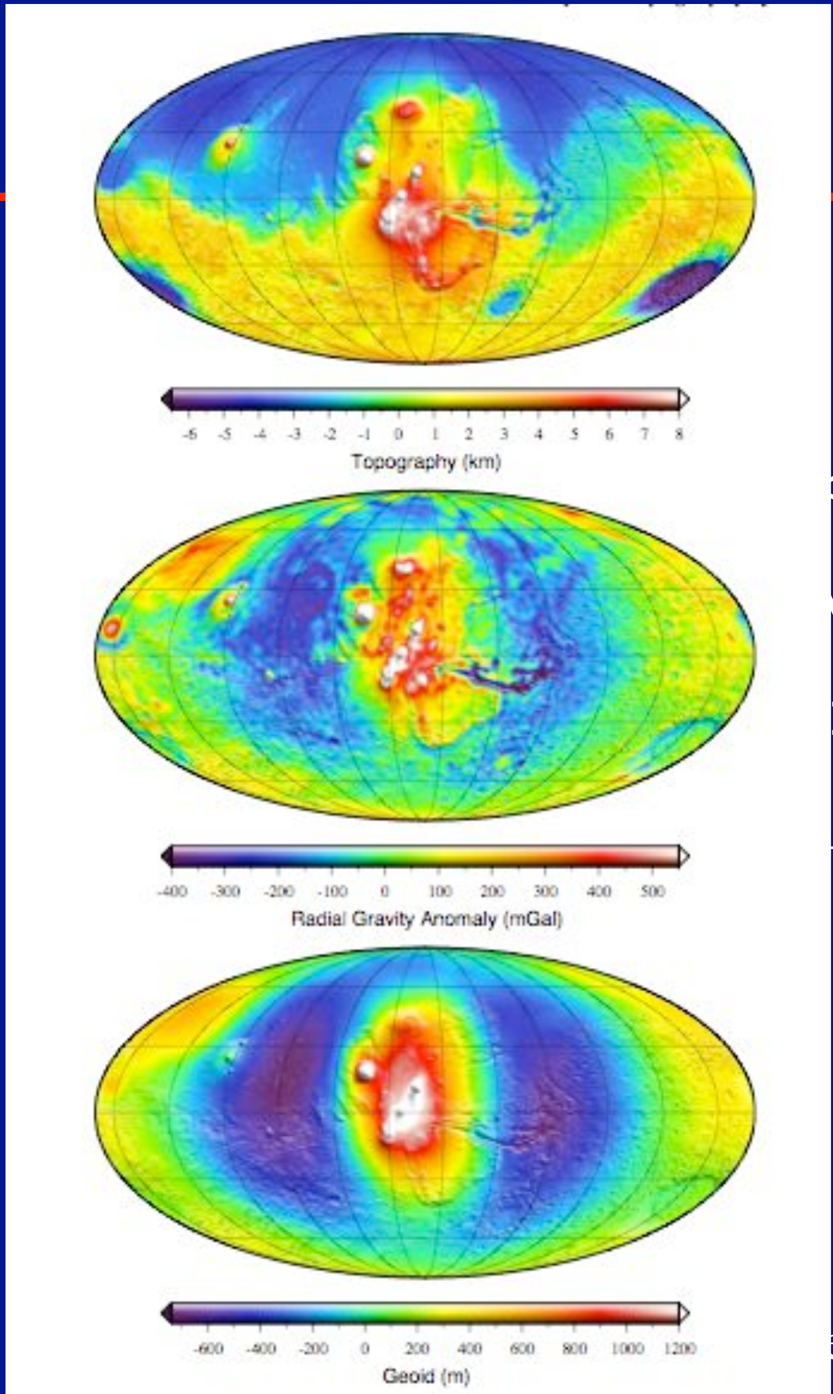
Terre

->

Venus



Wieczorek, 200



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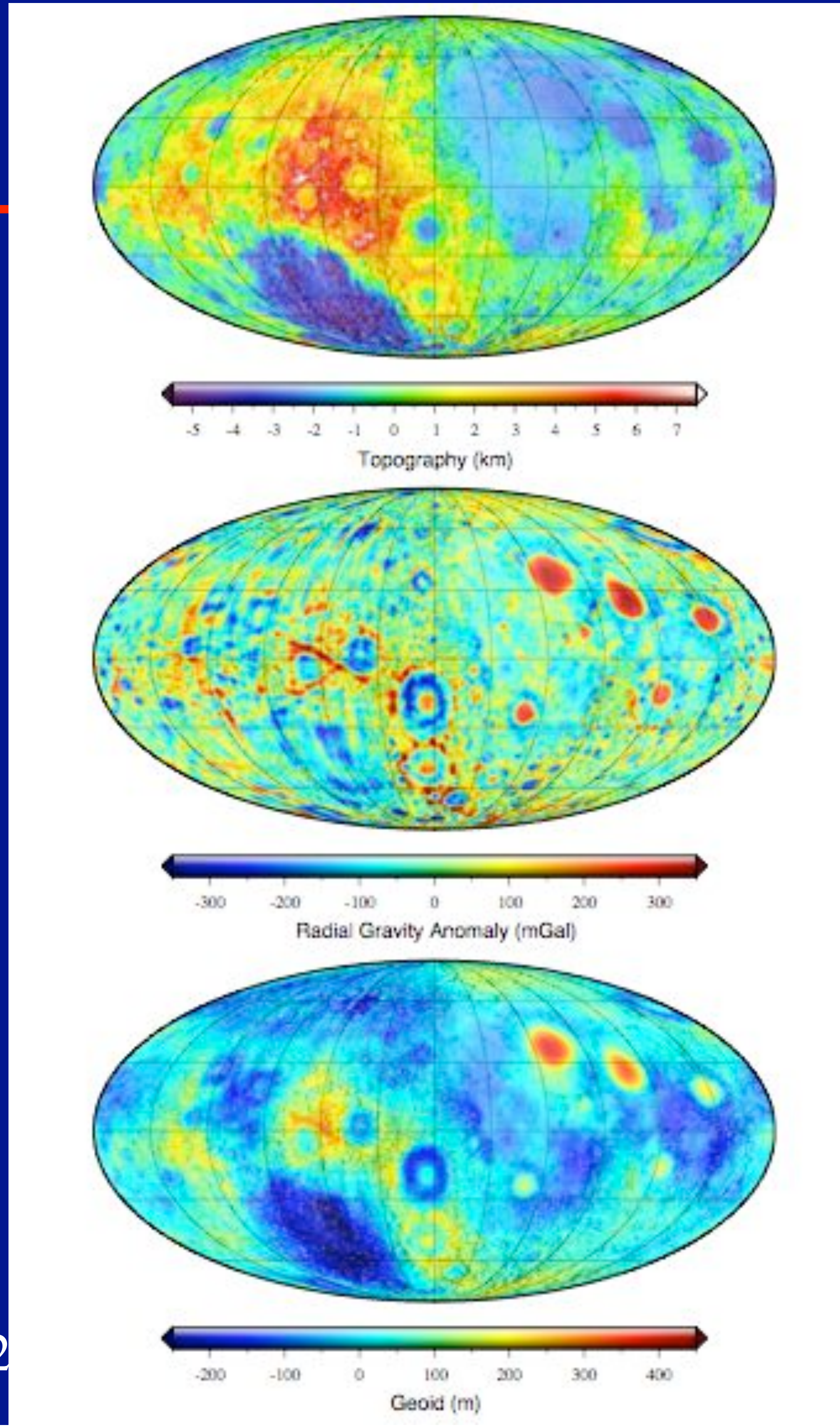
Mars

>

Lune

eczorek, 2



Non unicity of the gravity data:

Demonstrate that gravity data cannot provide the mean thickness of the crust!

Seismology and crust

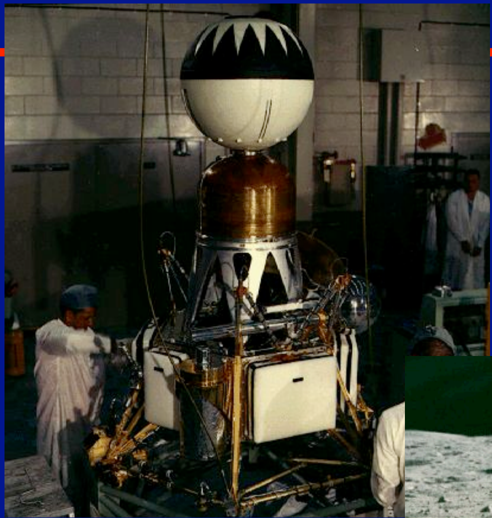
Moon

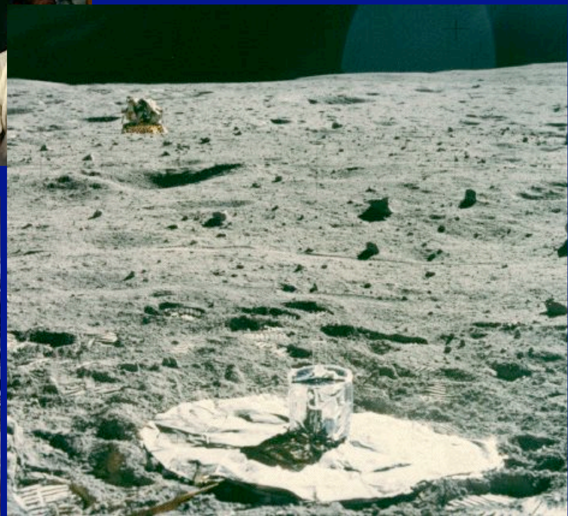
Other planets…(Europa)

Seismology outlines

- How to do Planetary seismology: the exemple of the Lunar case
- How to prepare future Mars seismic network mission
- Future seismology perspectives

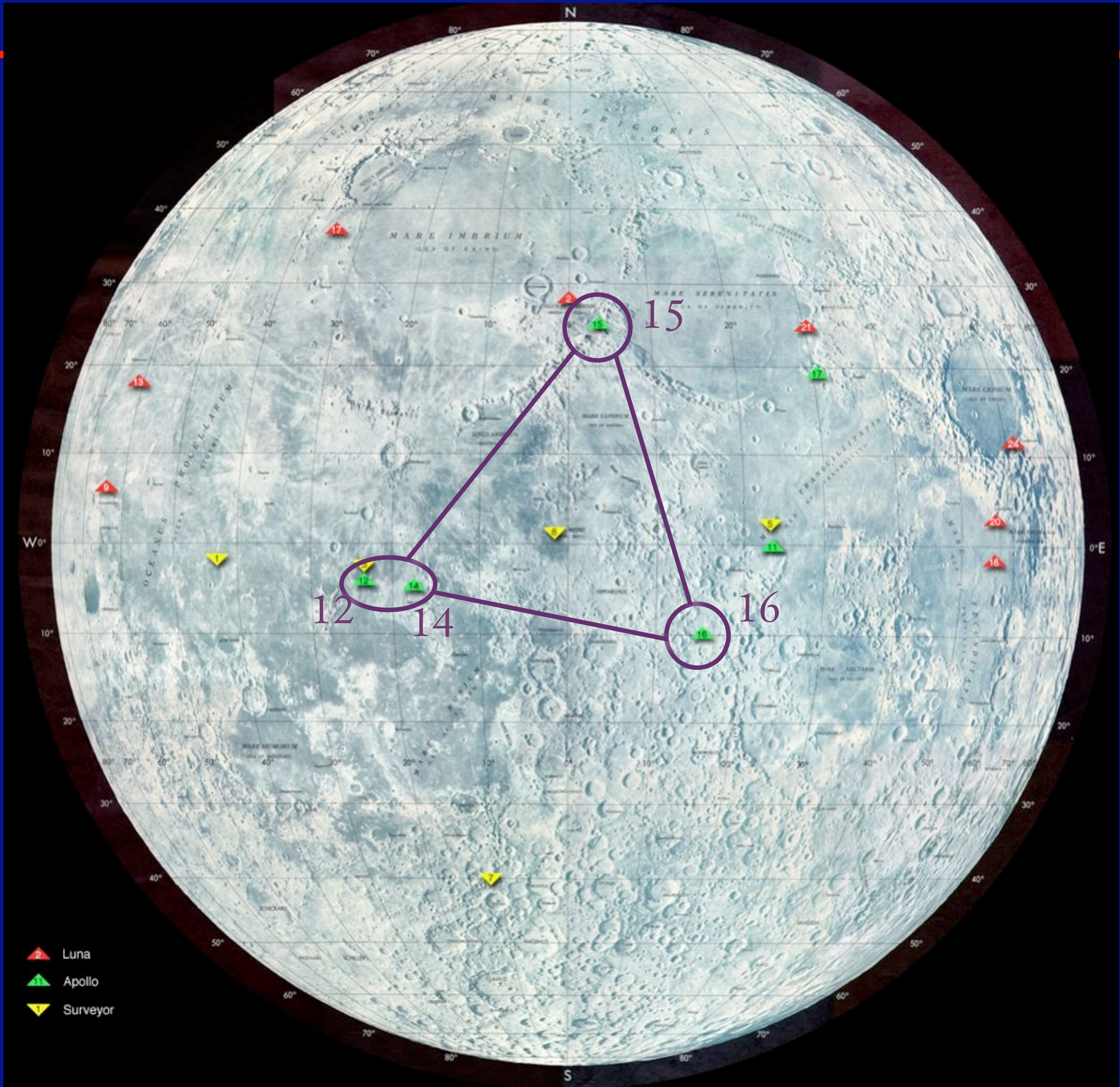
Moon Seismology: history





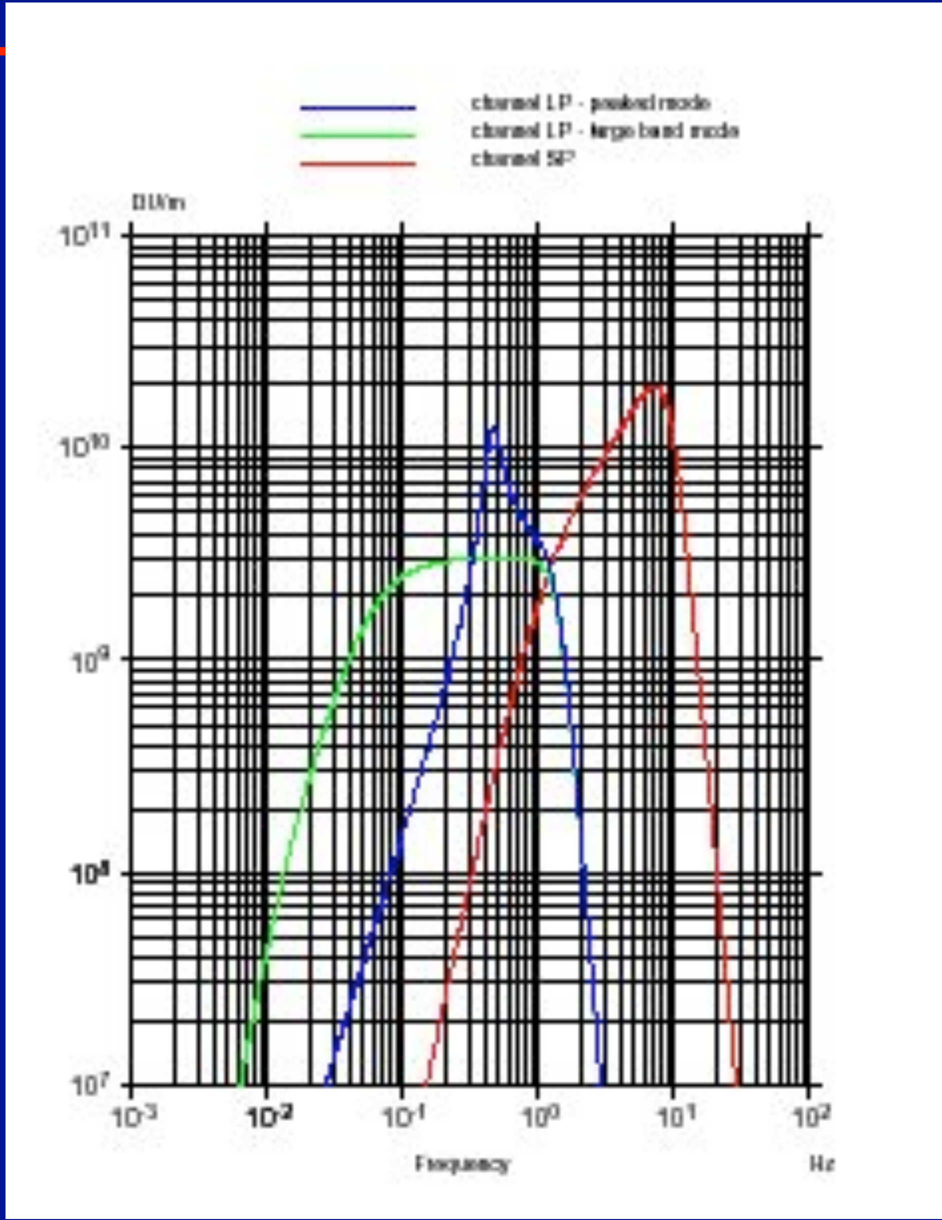
- 1961-1962: Failure of Rangers 3-5 all with seismometer launched to the Moon
- 1969-1973: Success of Apollo Seismic network with operation up to 1977
- 2006+: 2 antipodal seismic penetrators to be launched by Japan-ISAS (Lunar A)
- Other data for deep interior:
  - Density
  - Inertia factor
  - Love number (real and imaginary part)
  - Heat flux (2 measurements) + surface content in Th

Apollo Seismic Network

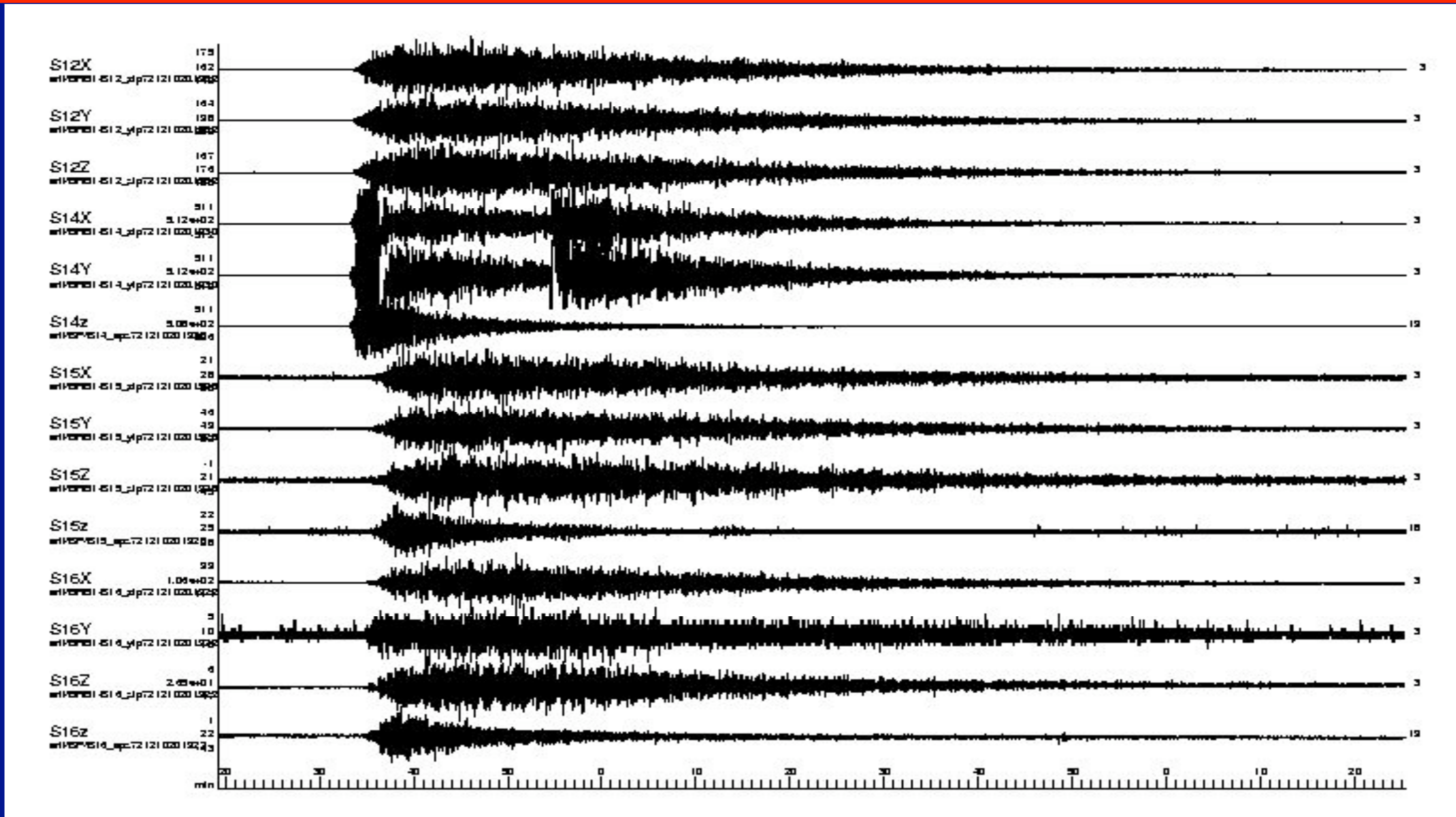


- 4 stations: Apollo sites 12, 14, 15 and 16
- installed between 1969 and 1972
- turned off in 1977

sensitivity



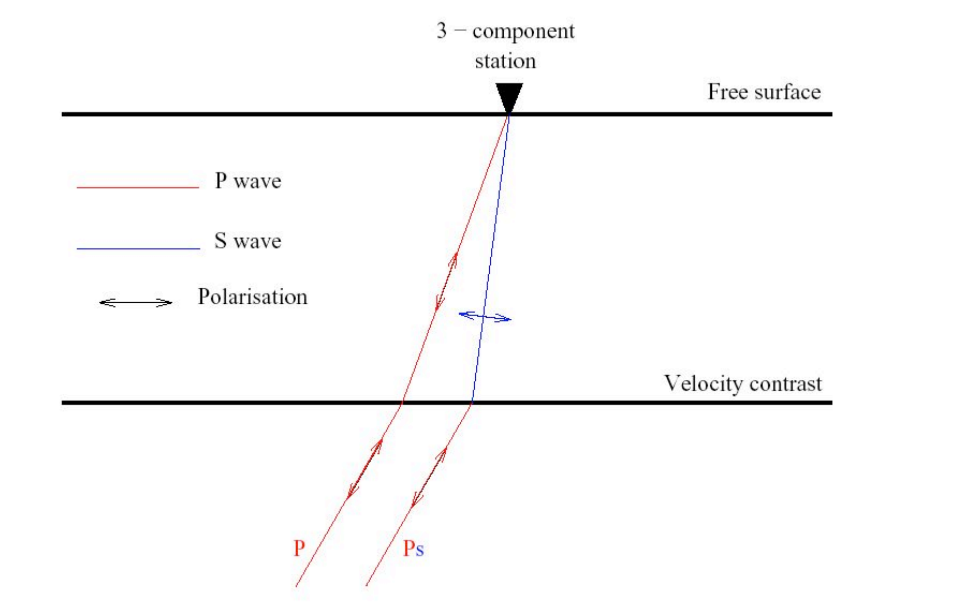
Active source: impacts

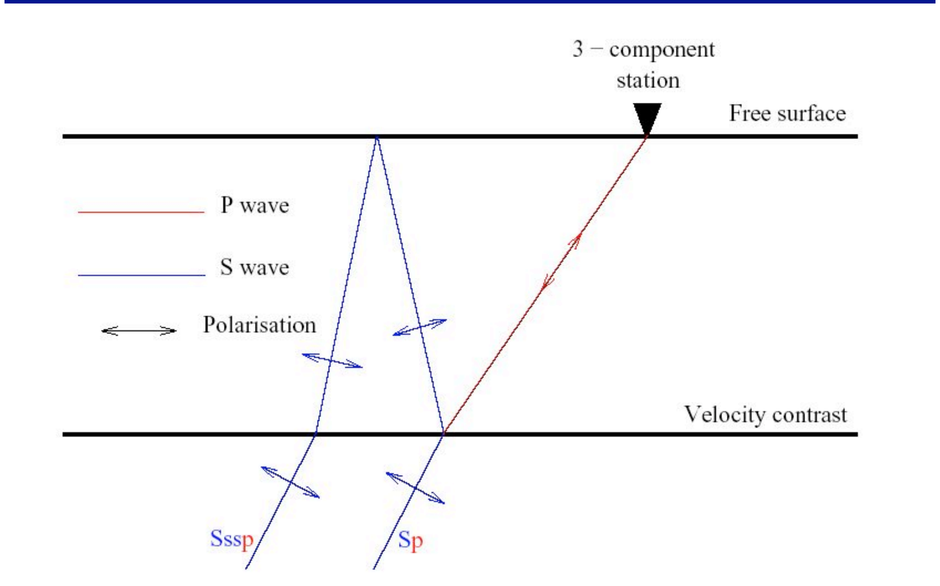


- Impact of the Apollo 17 Saturn V upper stage (Saturn IVB) on the Moon on 10 December 1972 at distances of 338, 157, 1032 and 850 km from the Apollo 12, 14, 15 and 16 stations, respectively. Amplitudes at Apollo 14 station, 157 km from impact, reach about 10^{-5} m s^{-2}
- Known time and location: all arrival times give information on the structure

Inversion with some 3D effects: crustal structure

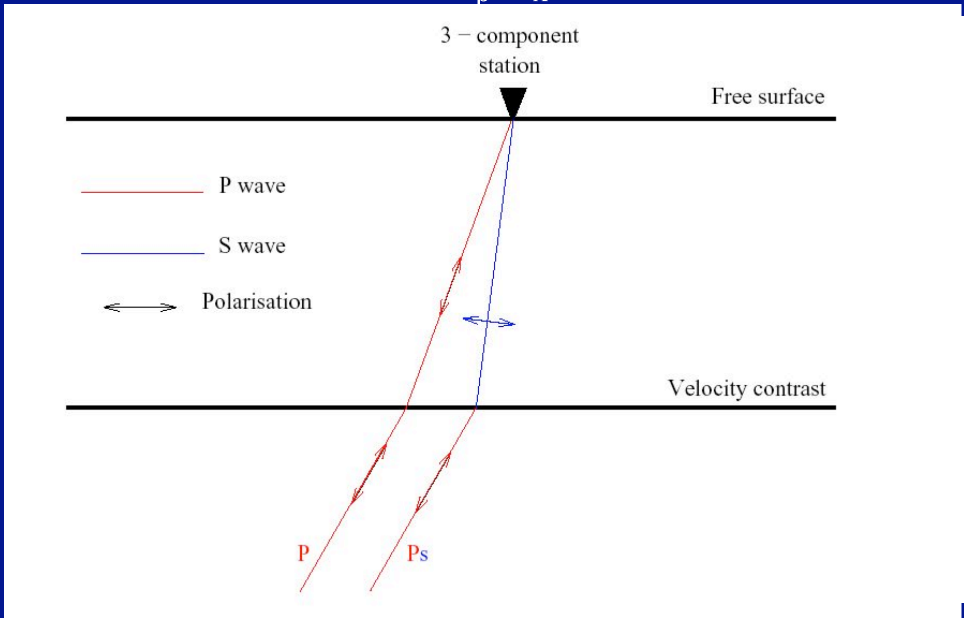
- The crustal structure leads to conversion and reverberations
  - Primary wave arrival ~$P(t-t_p)$ x T
    - P(t) is the amplitude in of the P wave below the crust, depending on the mantle propagation and of the seismic source, T the transmission coefficient to the crust and tp the transmission time through the crust
  - Converted wave ~$P(t-t_c)$ c C
    - C is the transmission coefficient of the crust from Primary wave to converted wave and tc the transmission time through the crust

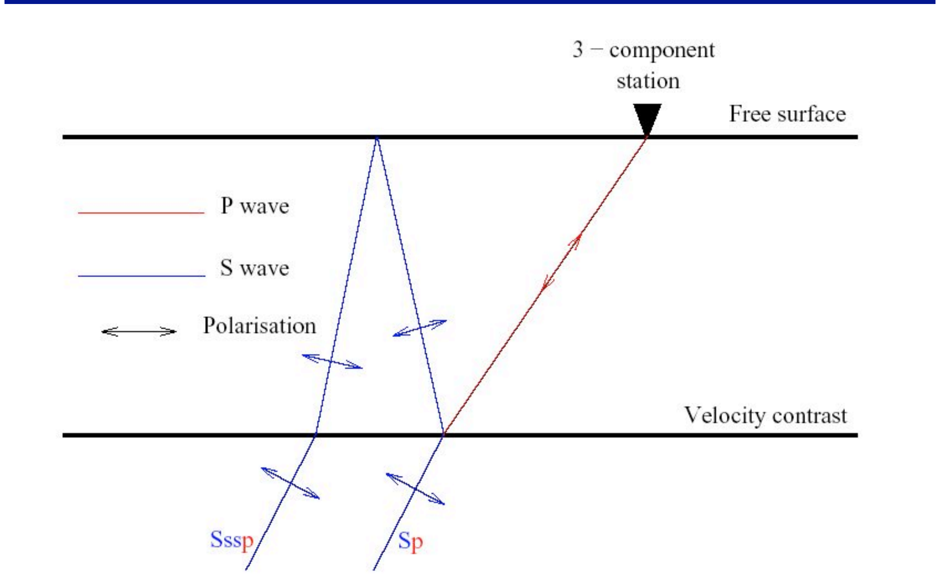




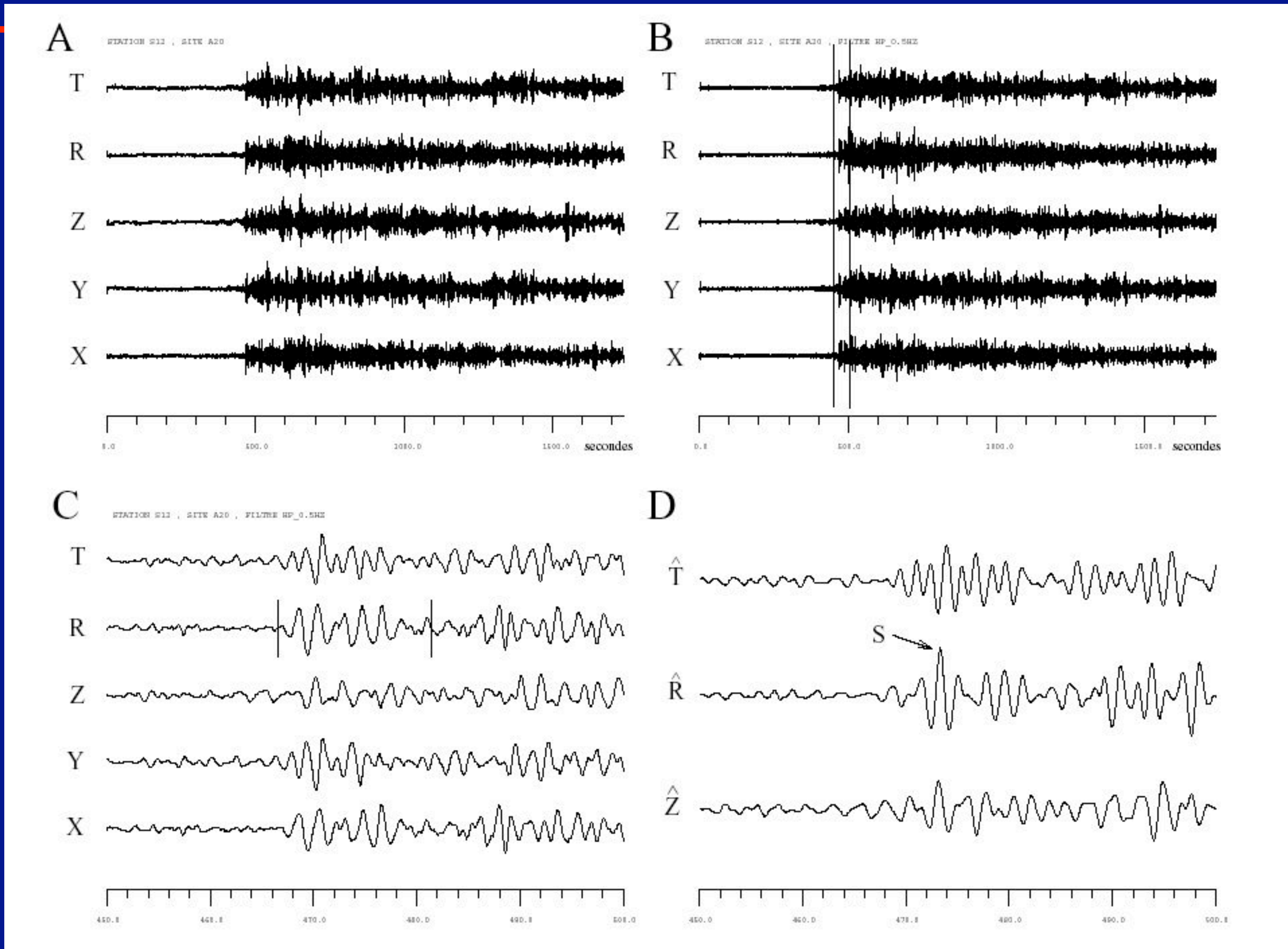
Receiver function method

- 1st step : make the Fourier transformation of the arrivals
  - Primary wave arrival Fourier Transformation ~ $T\,P(\omega)\exp(i\omega t_p)$
  - Converted wave ~$C\,P(\omega)\exp(i\omega t_x)$
- 2nd step: perform the deconvolution of the converted wave by the primary wave in frequency domain
  - $R(\omega)= [T\,P(\omega)\exp(i\omega t_p)] / [C\,P(\omega)\exp(i\omega t_x)] = T/C\exp(i\omega(t_p - t_x))$
- 3rd step: perform the inverse Fourier transformation
  - $R(t)=T/C\,\delta((t_p - t_x)$

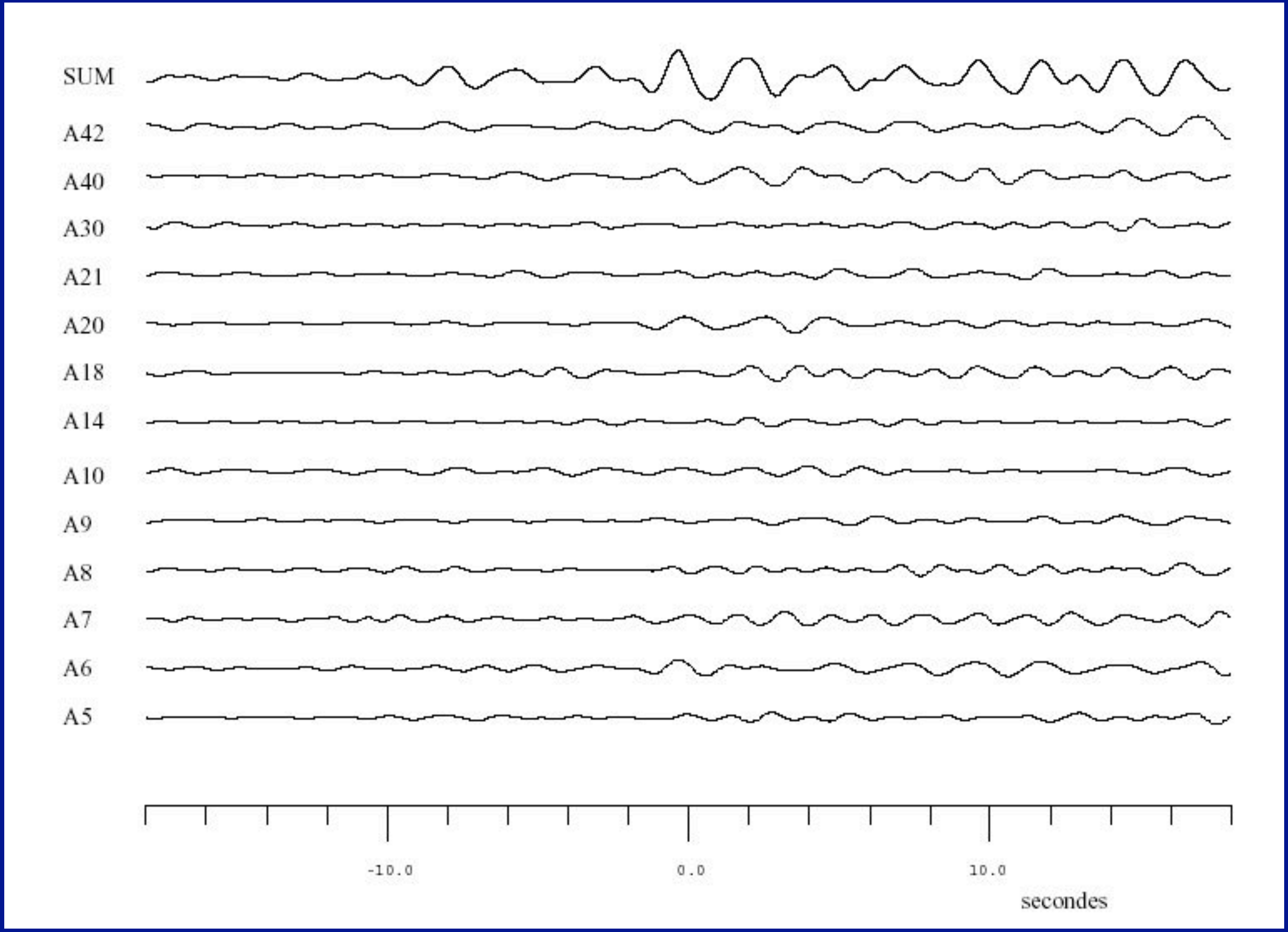




Deconvolution process



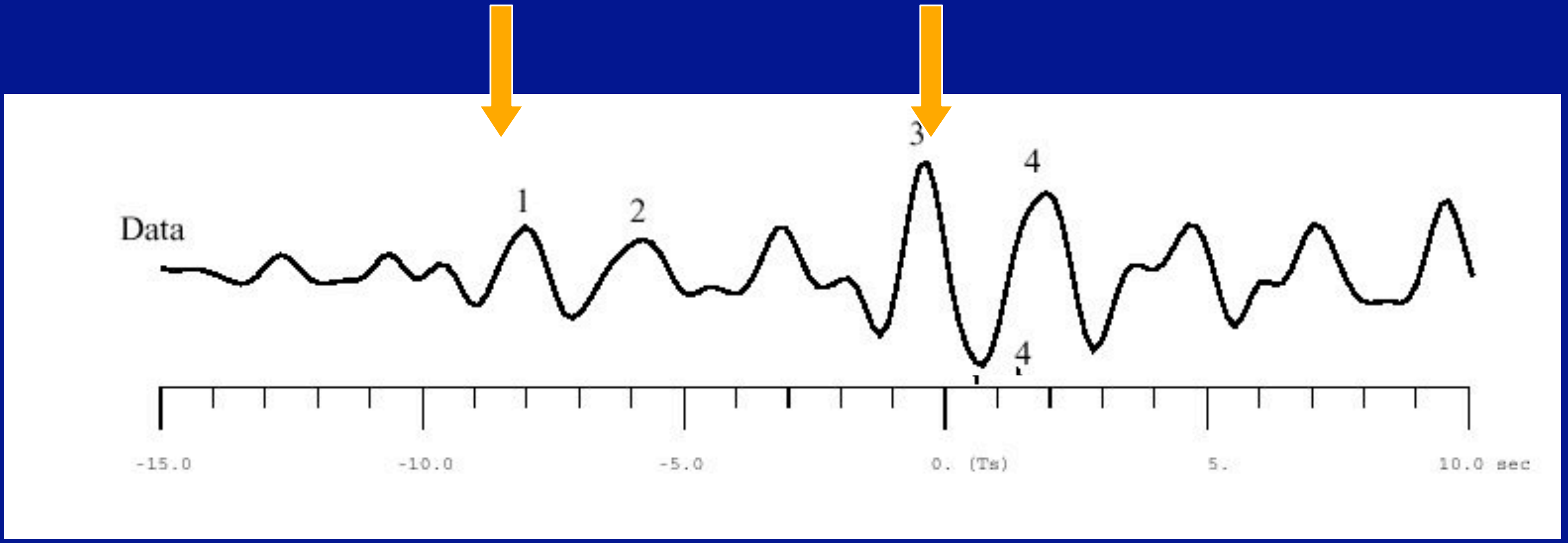
Improving the signal to noise ratio with stack



Moon receiver function (Apollo 12 site)

S->P conversion at the Base of the crust

Subsurface/regolith delay and reverberation

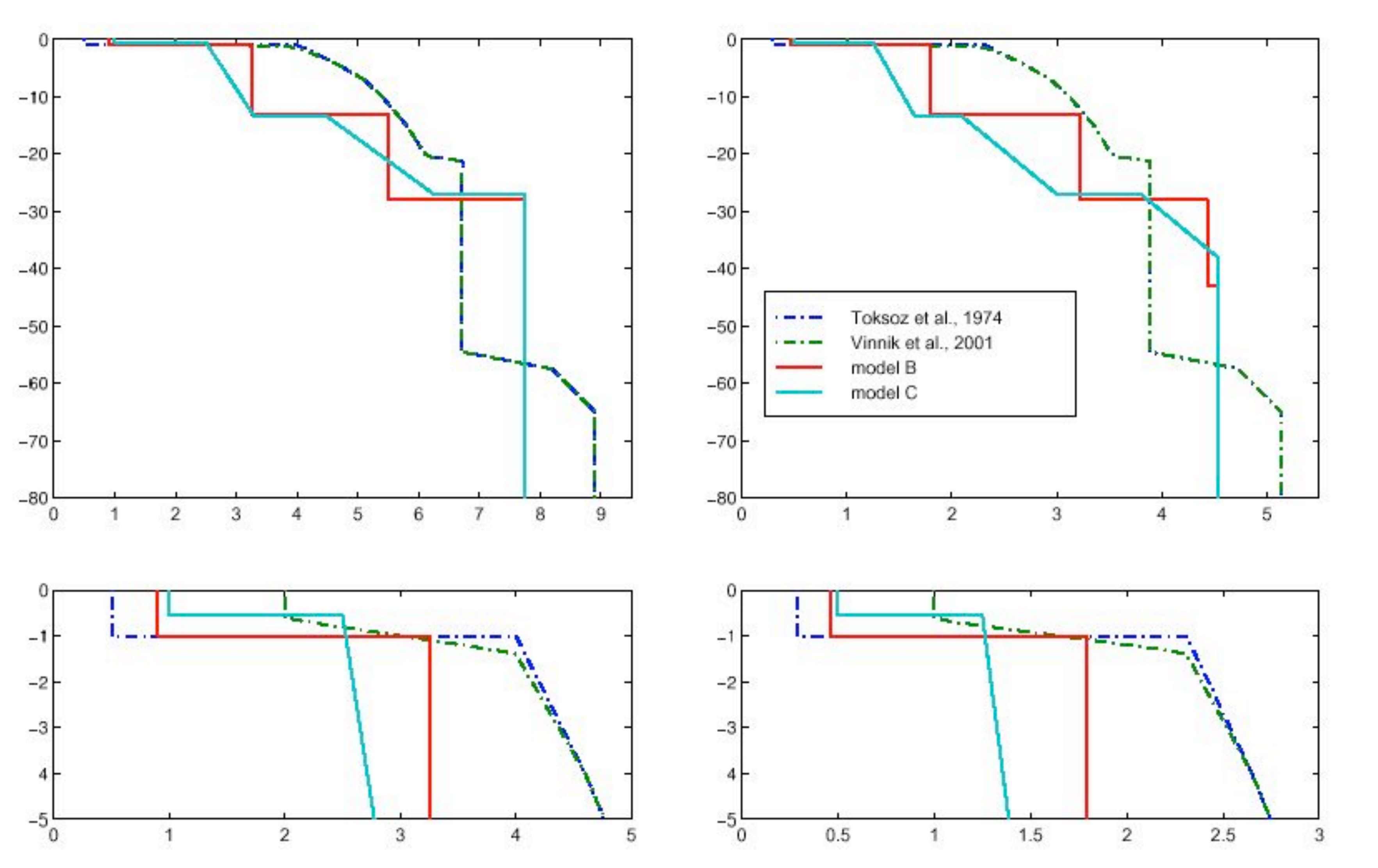


- S-P delay is equal to

$$\Delta t = t_s - t_p = D\left(\frac{1}{v_s} - \frac{1}{v_p}\right)$$

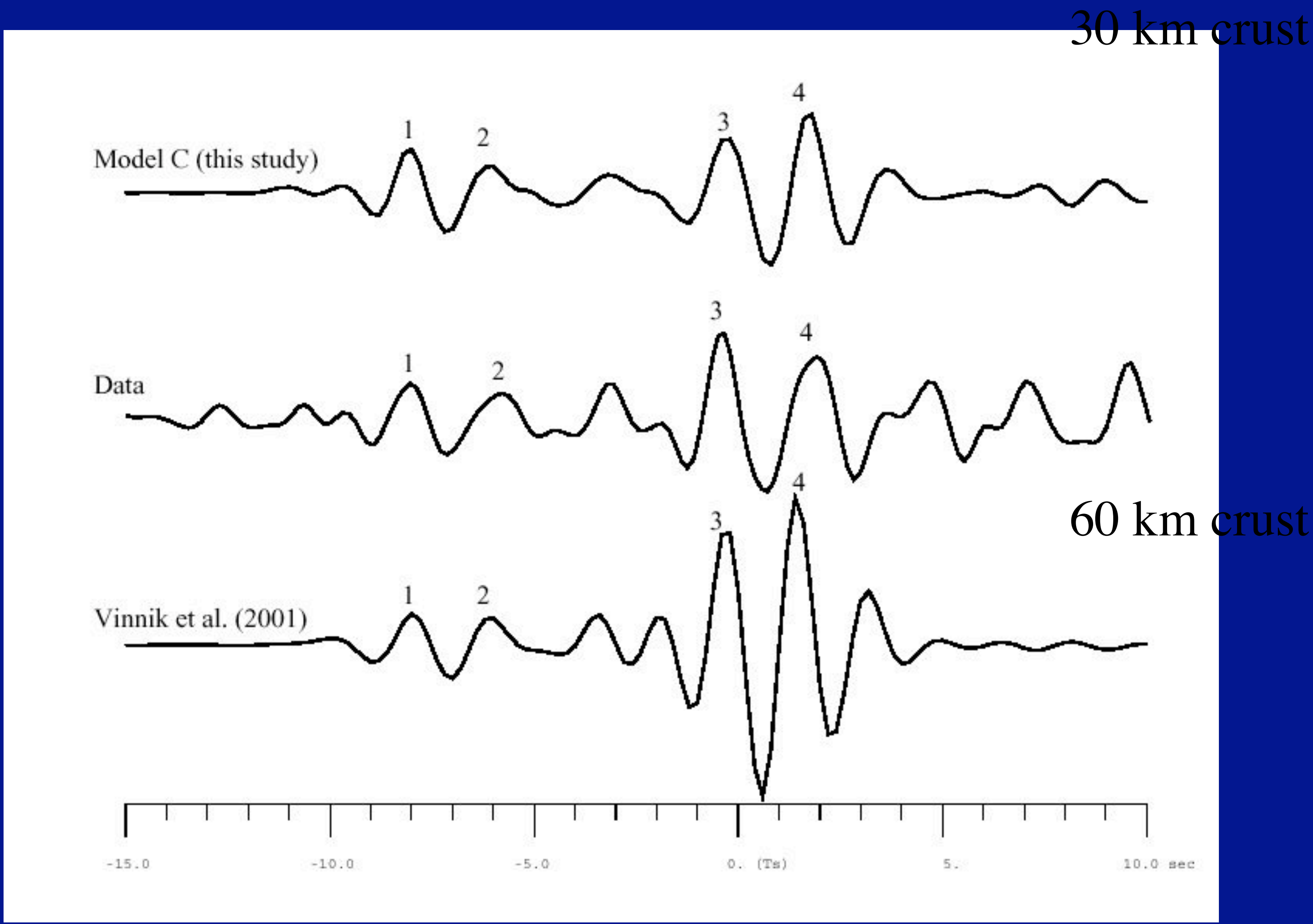
And therefore does not give a unique solution

- other informations are needed (amplitude conversion coefficient)

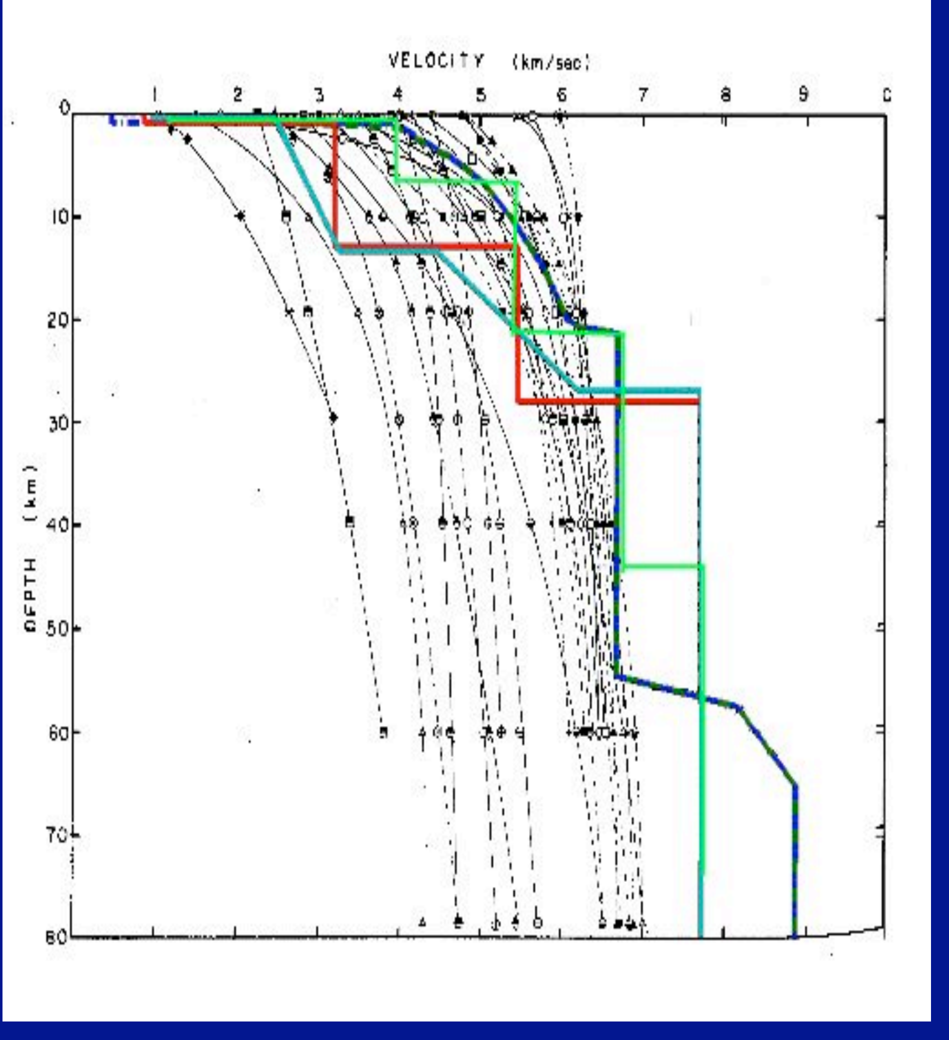


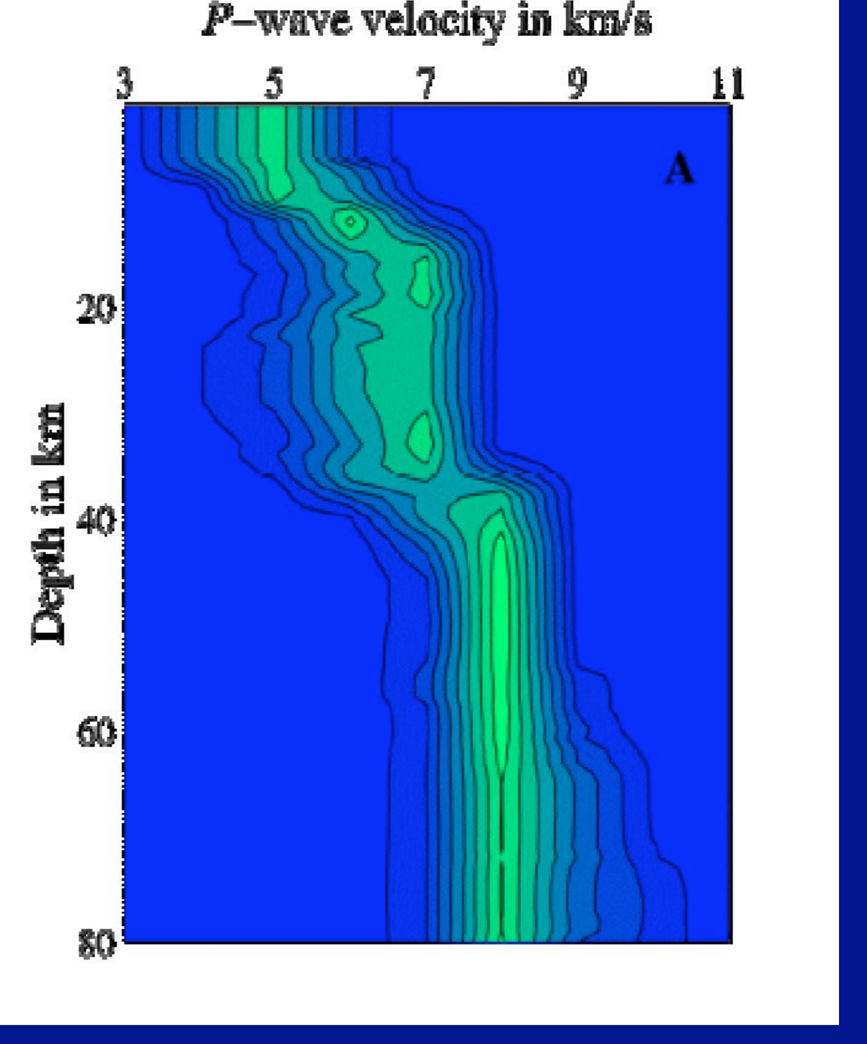
Toksoz et al., 1974
Vinnik et al., 2001
model B
model C
0
−10
−20
−30
−40
−50
−60
−70
−80
0 1 2 3 4 5 6 7 8 9
0 1 2 3 4 5
0
-1
-2
-3
4
-5
0 1 2 3 4 5
0 0.5 1 1.5 2 2.5 3

Moon receiver function (Apollo 12 site)



Crustal models





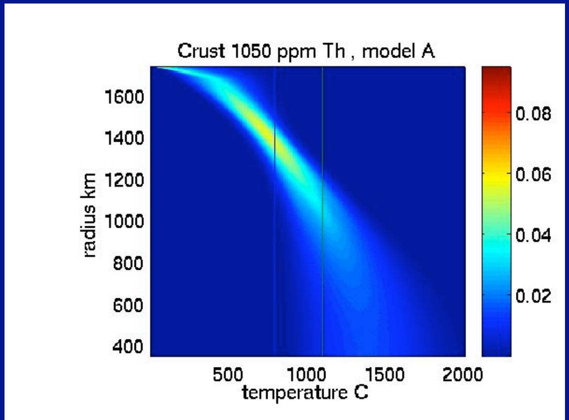
Gravity

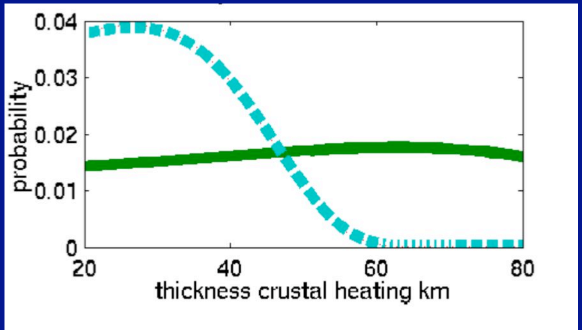
Seismology

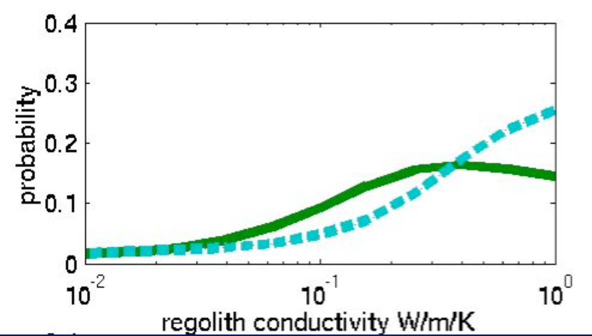
Interpretation of the seismic models

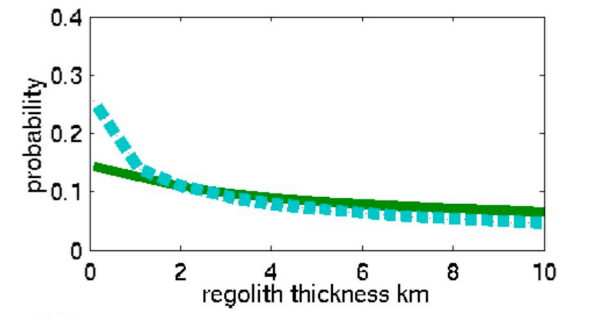
Temperature modeling

- Fit of seismic velocities for a known mineralogy
  - Seismic velocities are mainly a thermometer constraining the temperature
- Temperature model with regolith insulation, crustal heating, and upper/lower mantle heating









What Can Geophysics Tell Us?

The Crust:

1. What is its thickness and composition?
2. How and when did it form?
3. How has the crust responded to surface loads such as volcanoes?

Thermal Evolution:

1. How has the heat flow of Mars varied with time?

Polar Caps:

1. How much mass is transferred between the atmosphere and polar caps each year?

Did Mars Ever Possess and Ocean?

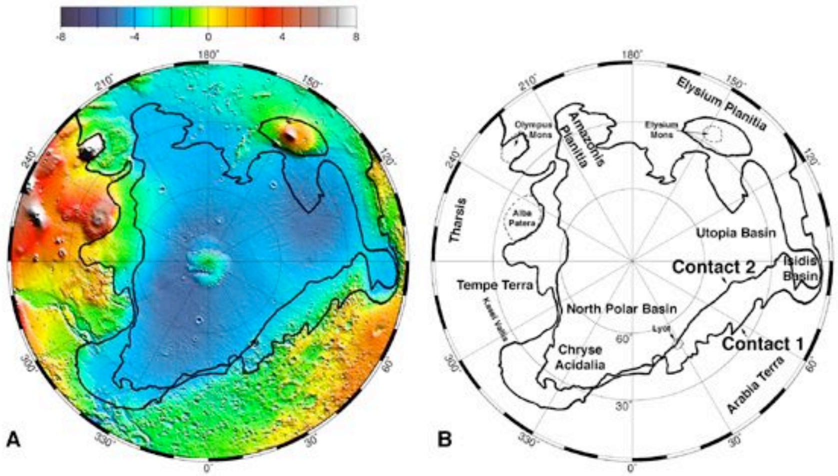


Figure 1

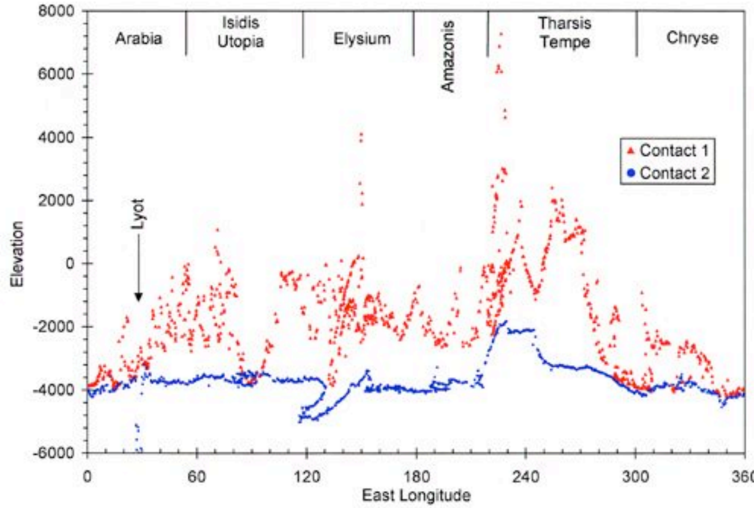


Figure 2

Head et al. (1999)

Parker et al. (1989, 1993) identified two quasi-continuous “contacts” in the northern hemisphere of Mars which they interpreted as being paleo-shorelines.

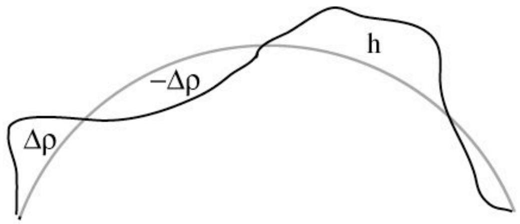
Using MGS MOLA and gravity data, *Head et al*. (1999) found that the inner contact closely approximated an equipotential surface, whereas the outer contact did not.

Has the gravity or topography of Mars changed since the putative ocean disappeared? Has the rotation axis changed?

The Relationship between Gravity and Topography

Approximate topography by a "sheet-mass"

ρ(r,θ,φ) = σ(r0,θ,φ) δ(r,r0) ~ Δρ h(θ,φ) δ(r,r0)



Expand σ and h into spherical harmonics

$$\sigma(r_0,\theta,\phi) = \sum_{i=1}^{2}\sum_{l=0}^{l_{max}}\sum_{m=0}^{l} \sigma_{ilm} Y_{ilm}(\theta,\phi) \qquad h(\theta,\phi) = \sum_{i=1}^{2}\sum_{l=0}^{l_{max}}\sum_{m=0}^{l} h_{ilm} Y_{ilm}(\theta,\phi)$$

Insert into

$$C_{ilm} = \frac{1}{(2l+1)MR^l}\int_V r^l \rho(r,\theta,\phi)\, Y_{ilm}(\theta,\phi)\, dV$$

using othogonality of spherical harmonics yields

$$C_{ilm} = \frac{4\pi r_0^{\,l}}{(2l+1)MR^l}\sigma_{ilm}$$

or to first order

$$C_{ilm} = \frac{4\pi \Delta\rho\, r_0^{\,l}}{(2l+1)MR^l} h_{ilm}$$

Gravity Modeling is Not Unique!

$$C_{ilm} = \frac{4\pi r_0^{\,l}}{(2l+1)MR^l}\sigma_{ilm}$$

What does this equation mean?

1. Any gravity field can be interpreted as being the result of a sheet mass with density variations σ_{ilm}.
2. The magnitude of σ_{ilm} depends upon the radius, r_0, that the sheet mass is placed at.

Gravity modeling by itself is inherently non-unique because only the product r^l_o σ_{ilm} can be determined from a given gravity field.

How do you get around this non-uniqueness problem in geophysics?

One must make assumptions such as:

1. The density of the crust and mantle are uniform
2. Surface topography is compensated by Airy, Pratt or flexure
3. The average thickness of the crust is known.

How Good is the "mass-sheet" Approximation?

The mass-sheet approximation is routinely used with terrestrial data. However since the dynamic range of topography on Mars is much larger (-8 to 30 km), is this still a valid approximation?

Taking into account the "finite-amplitude" of surface topography, its corresponding potential can be computed using the equation (*Wieczorek and Phillips*, 1998)

$$C_{ilm} = \frac{4\pi \Delta\rho D^3}{M(2l+1)} \sum_{n=1}^{n_{max}} \frac{{}^{n}h_{ilm}}{D^n n!} \frac{\prod_{j=1}^{n}(l+4-j)}{(l+3)}$$

where ${}^{n}h_{ilm}$ is the topography raised to the n^{th} power expanded in spherical harmonics.

n	Maximum Error in Computed Gravity Field
1 (mass-sheet approximation)	591 mgals
2	97 mgals
3	14 mgals
4	2 mgals
5	0.2 mgals
6	0.02 mgals

Maximum gravity anomaly on Mars ~2000 mgals

Maximum uncertainty of martian gravity field ~7 mgals.

Quelques propriétés

$$C_{\ell m} = \frac{1}{4\pi}\int U(\Omega)Y_{\ell m}(\Omega)d\Omega$$

$$D_{\ell m} = \frac{1}{4\pi}\int V(\Omega)Y_{\ell m}(\Omega)d\Omega$$

$$\frac{1}{4\pi}\int U^2(\Omega)\ d\Omega = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{m=+\ell} C^2{}_{\ell m} = \sum_{\ell=0}^{\infty} S_{UU}(\ell)$$

$$S_{uu}(\ell) = \sum_{m=-\ell}^{m=+\ell} C^2{}_{\ell m}$$

$$\frac{1}{4\pi}\int U\ (\Omega)\ V\ (\Omega)d\Omega = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{m=+\ell} C_{\ell m} D_{\ell m} = \sum_{\ell=0}^{\infty} S_{UV}(\ell)$$

$$S_{UV}(\ell) = \sum_{m=-\ell}^{m=+\ell} C_{\ell m} D_{\ell m}$$

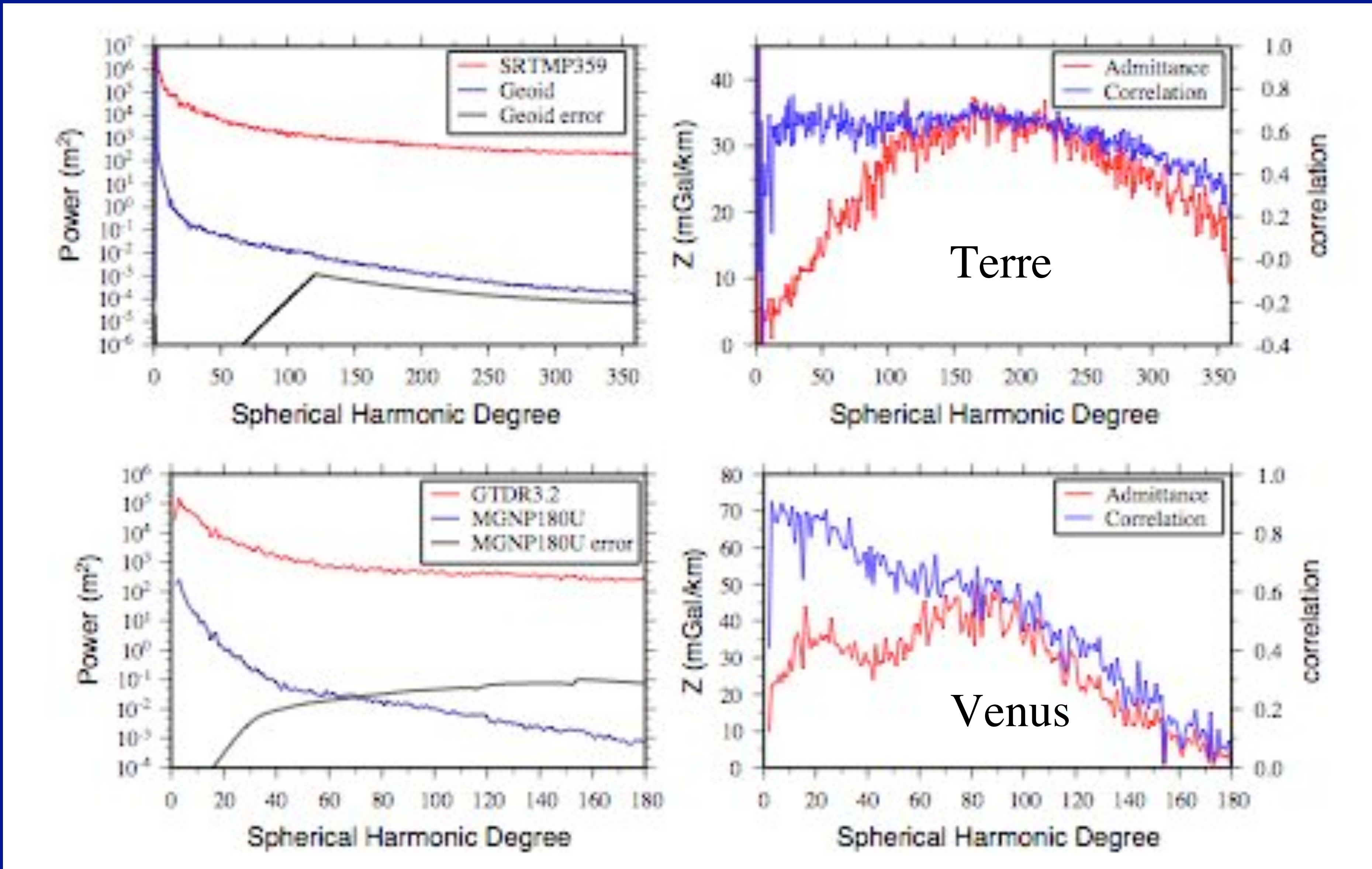
Quelques définitions

- Admittance et corrélation

$$Z(\ell) = \frac{S_{hg}(\ell)}{S_{hh}(\ell)}$$

$$\gamma(\ell) = \frac{S_{hg}(\ell)}{\sqrt{S_{hh}(\ell)}\sqrt{S_{gg}(\ell)}}$$

- Que valent ces deux valeurs si $g_{lm} = Q_\ell h_{\ell m}$?
- Que valent ces deux valeurs pour une approximations de couche mince?


SRTMP359
Geoid
Geoid error
Power (m²)
Spherical Harmonic Degree
Admittance
Correlation
Z (mGal/km)
correlation
Terre
GTDR3.2
MGNP180U
MGNP180U error
Venus