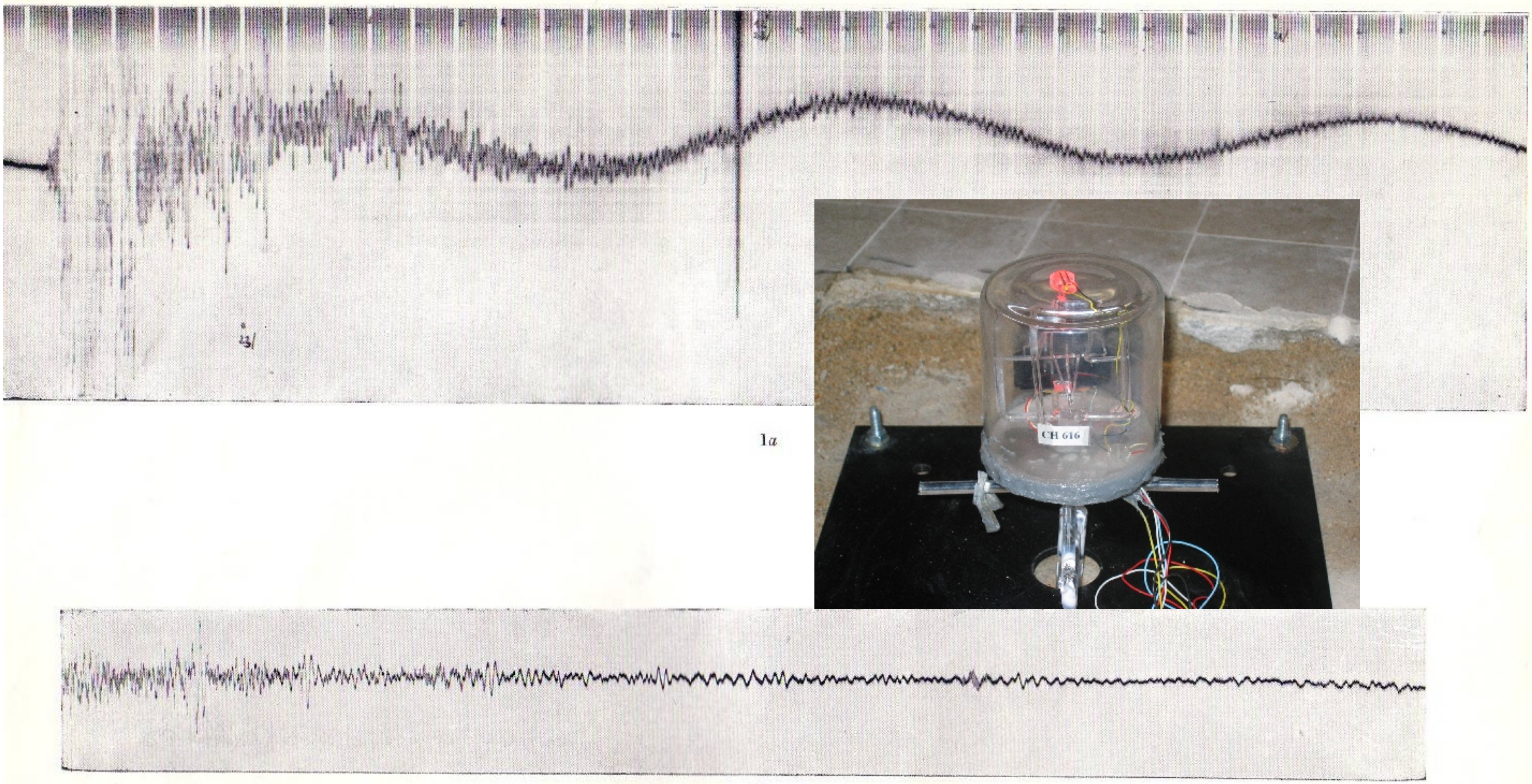
3rd class: Normal modes

-Observations

-Calculation of synthetic Seismograms

Chile Earthquake (22 may 1960) recorded at Paris (IPGP)



1*a*

1*b*

FIG. 1. — *a*) Enregistrement du pendule E, nº 1 (voir tableau I).
b) Enregistrement du pendule B, nº 4 (voir tableau I). (Un intervalle de 5 minutes est représenté sur cette figure par 1,45 mm.)

Chile earthquake (may 22 1960) recorded at Paris (IPGP)

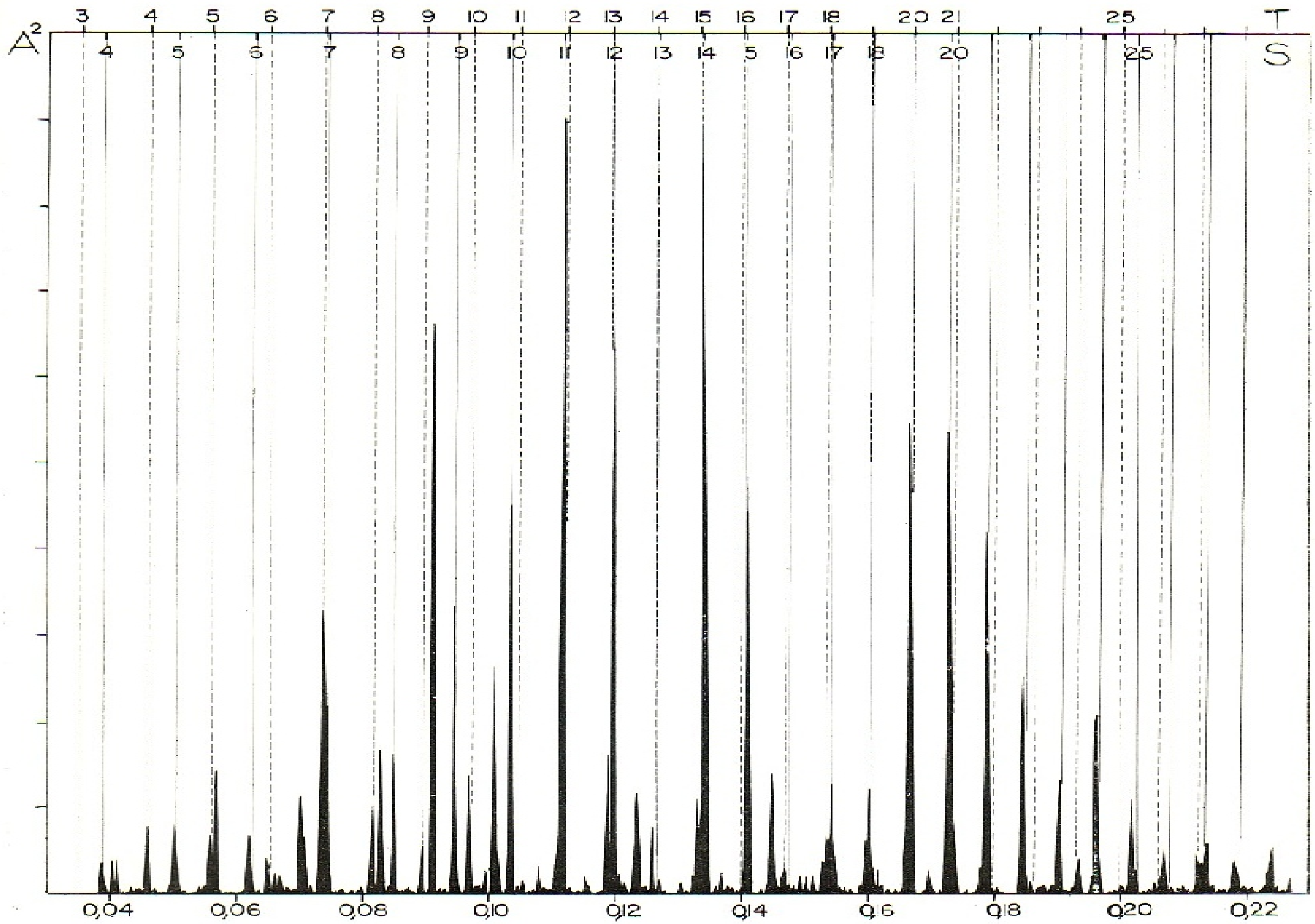
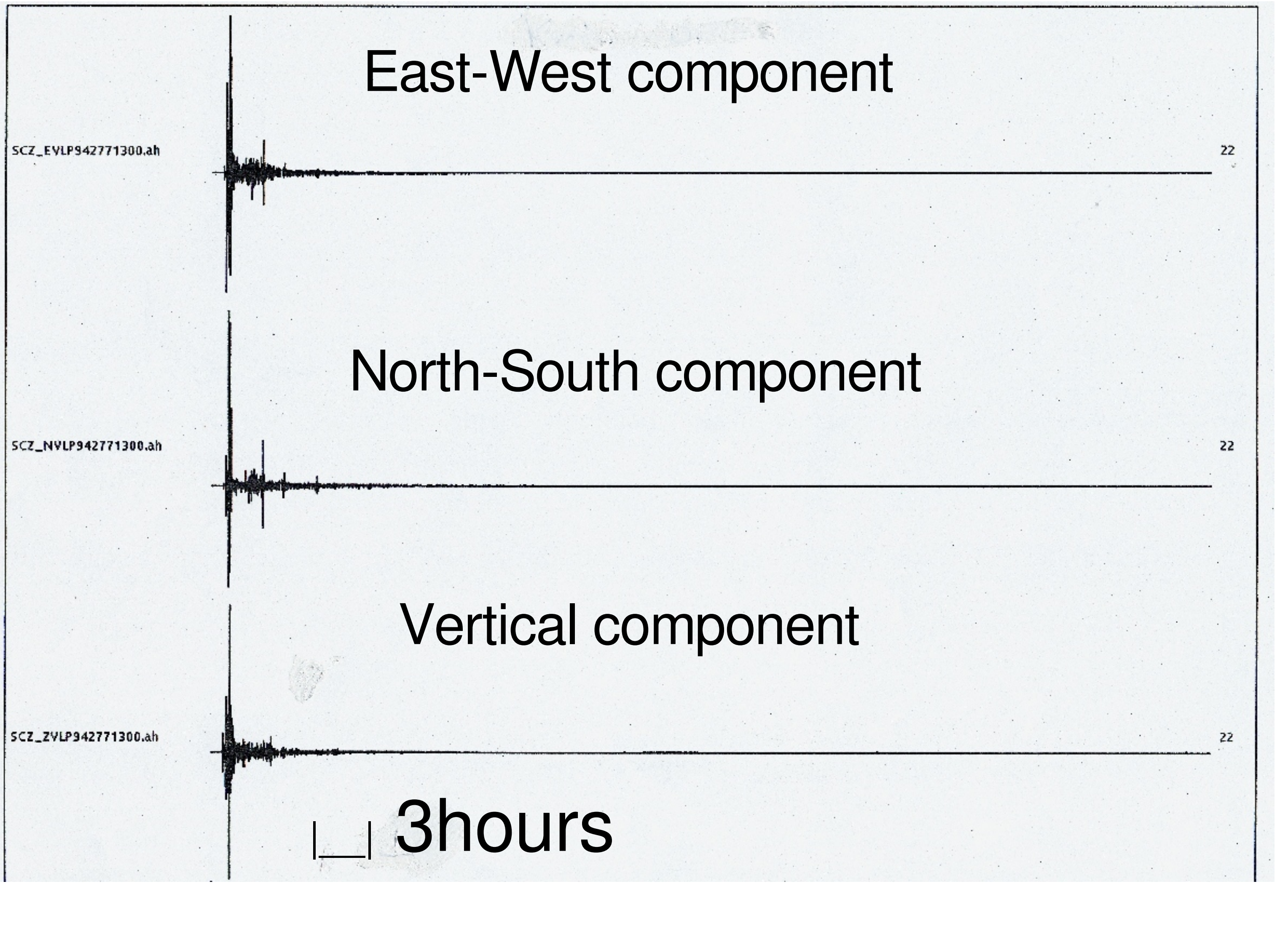
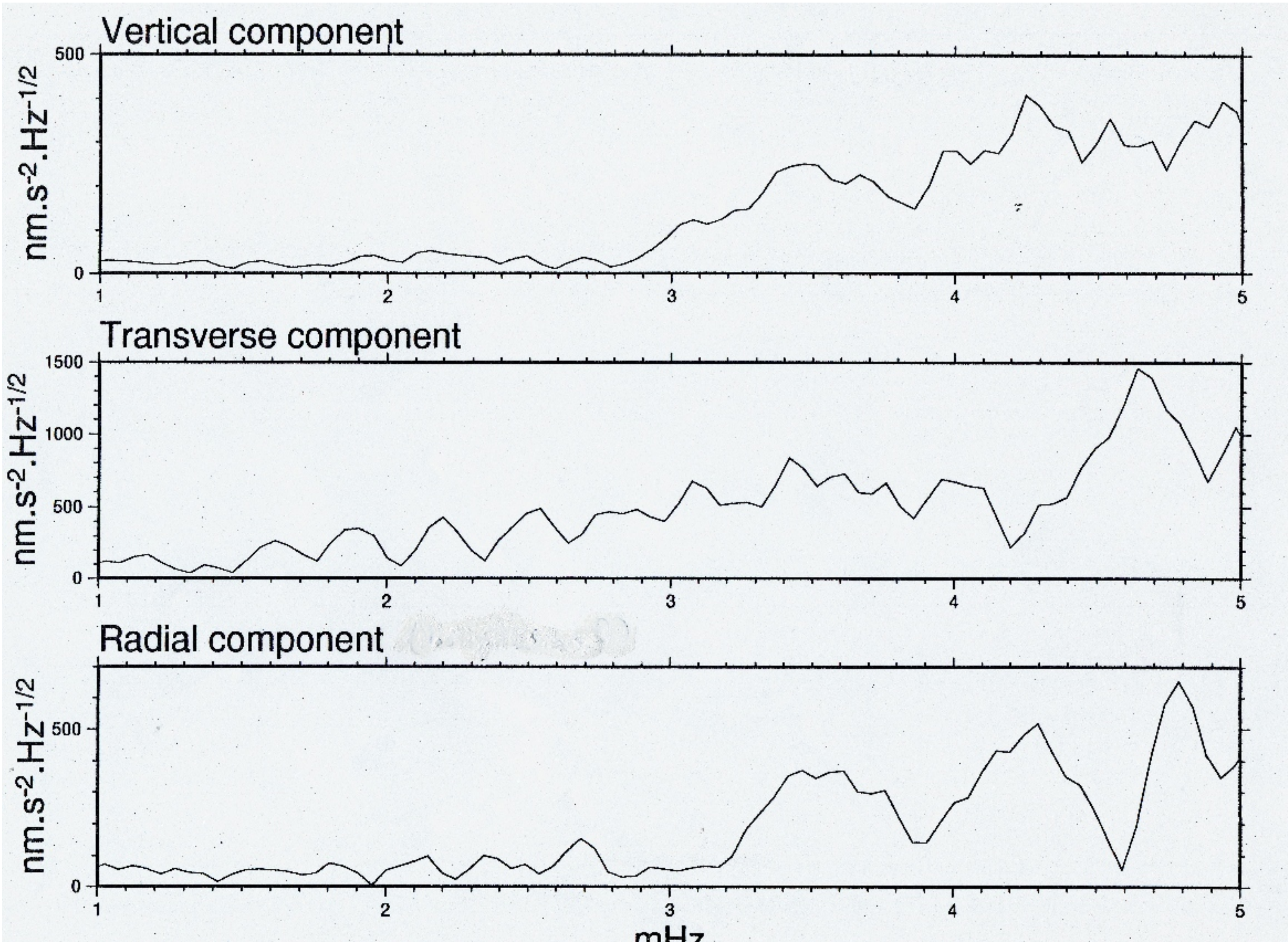


FIG. 3. — Spectre de l'enregistrement du pendule E (n° 1) — En haut : positions des pics théoriques pour les oscillations sphéroïdales S et les oscillations de torsion T du modèle de Gutenberg continental.

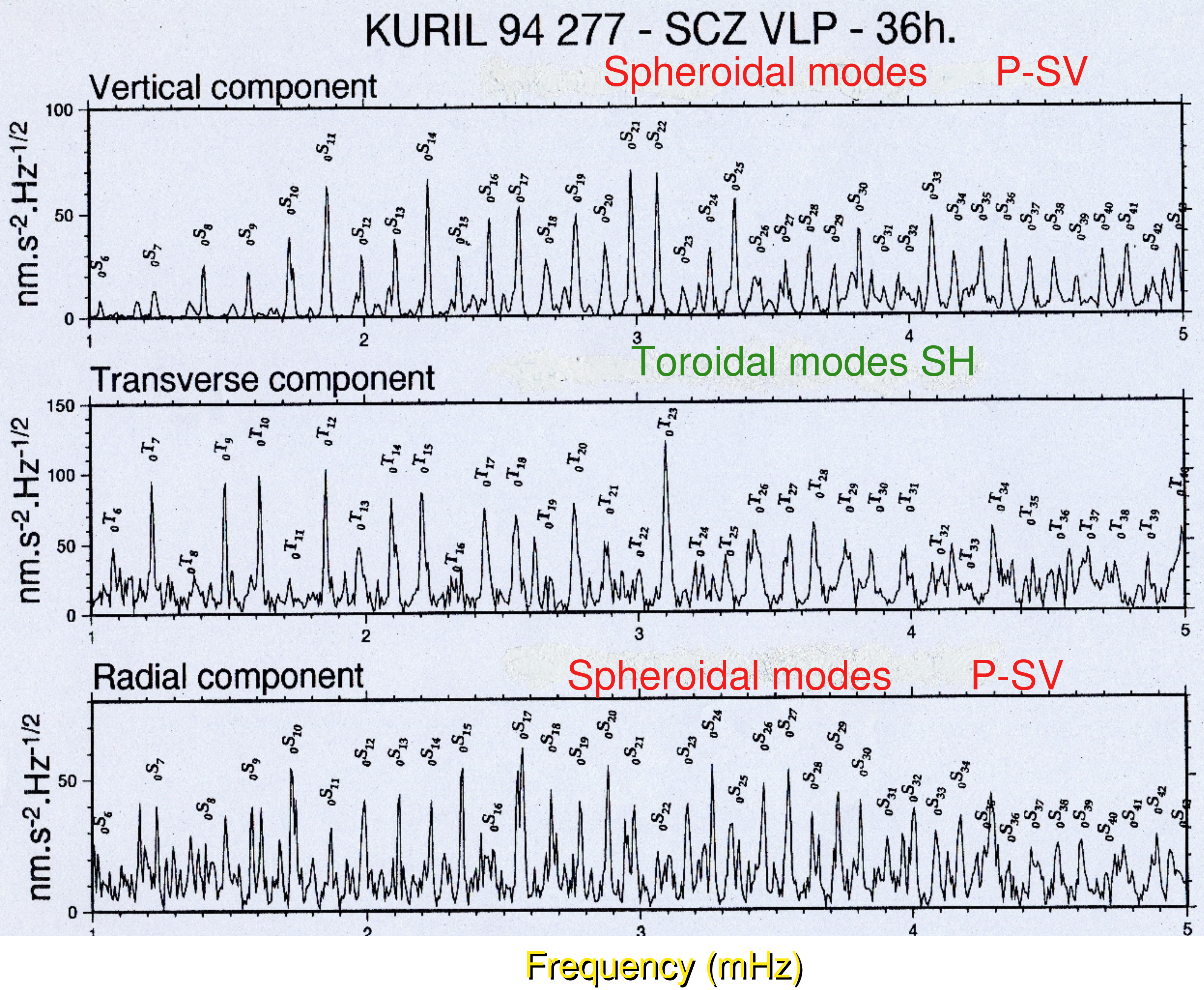
Kurils islands 1994-277 Ms=8.3



Kurils islands 1994-277 SCZ-VLP
Spectra 3 hours



Frequency (mHz)



KURIL 94 277 - SCZ VLP - 36h.
Spheroidal modes P-SV
Vertical component
Toroidal modes SH
Transverse component
Spheroidal modes P-SV
Radial component
nm.s-2.Hz-1/2
Frequency (mHz)

Elasto-dynamic equation

$$\rho\partial_{tt}\mathbf{u}_{0i} = \partial_j\boldsymbol{\sigma}_{\mathbf{ij}} + \rho\ \mathbf{g}_\mathbf{i} + \mathbf{F}_\mathbf{i}\ (+\ \mathbf{Fs}_\mathbf{i}\ +\ldots)$$

Which can be rewritten:

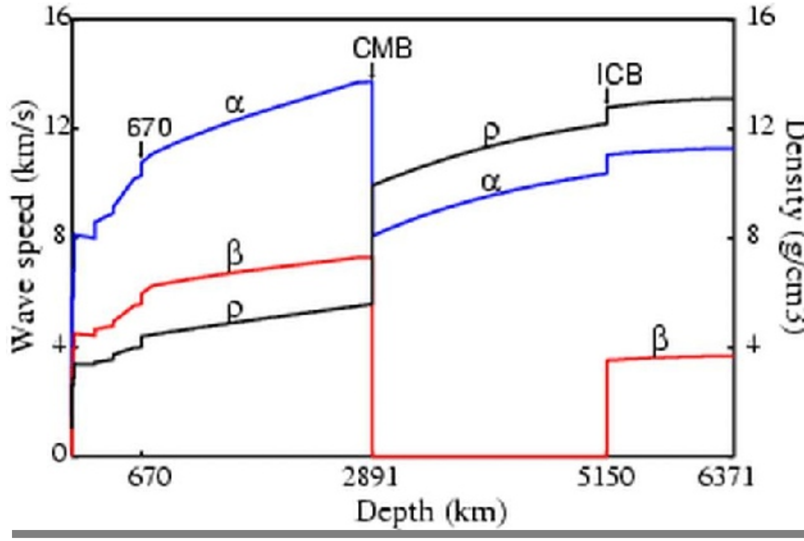
$$\rho\partial_{tt}\mathbf{u}_0 = \mathbf{H}_\mathbf{0}\mathbf{u}_\mathbf{0}\ \ (+\ \mathbf{Fs})$$

H_0 is an integro-differential operator

1D-Reference Earth Model:

$M_0(r)$, $\rho(r\ )$, $V_P(r)$, $V_s(r)$

(PREM, Dziewonski and Anderson, 1981
or IASP91, Kennett and Engdahl, 1991)



Eigenfrequencies: ${}_n\omega_l$

Eigenfunctions: ${}_nu_l^m\ (r,t) = |n,l,m>$

Spheroidal Modes

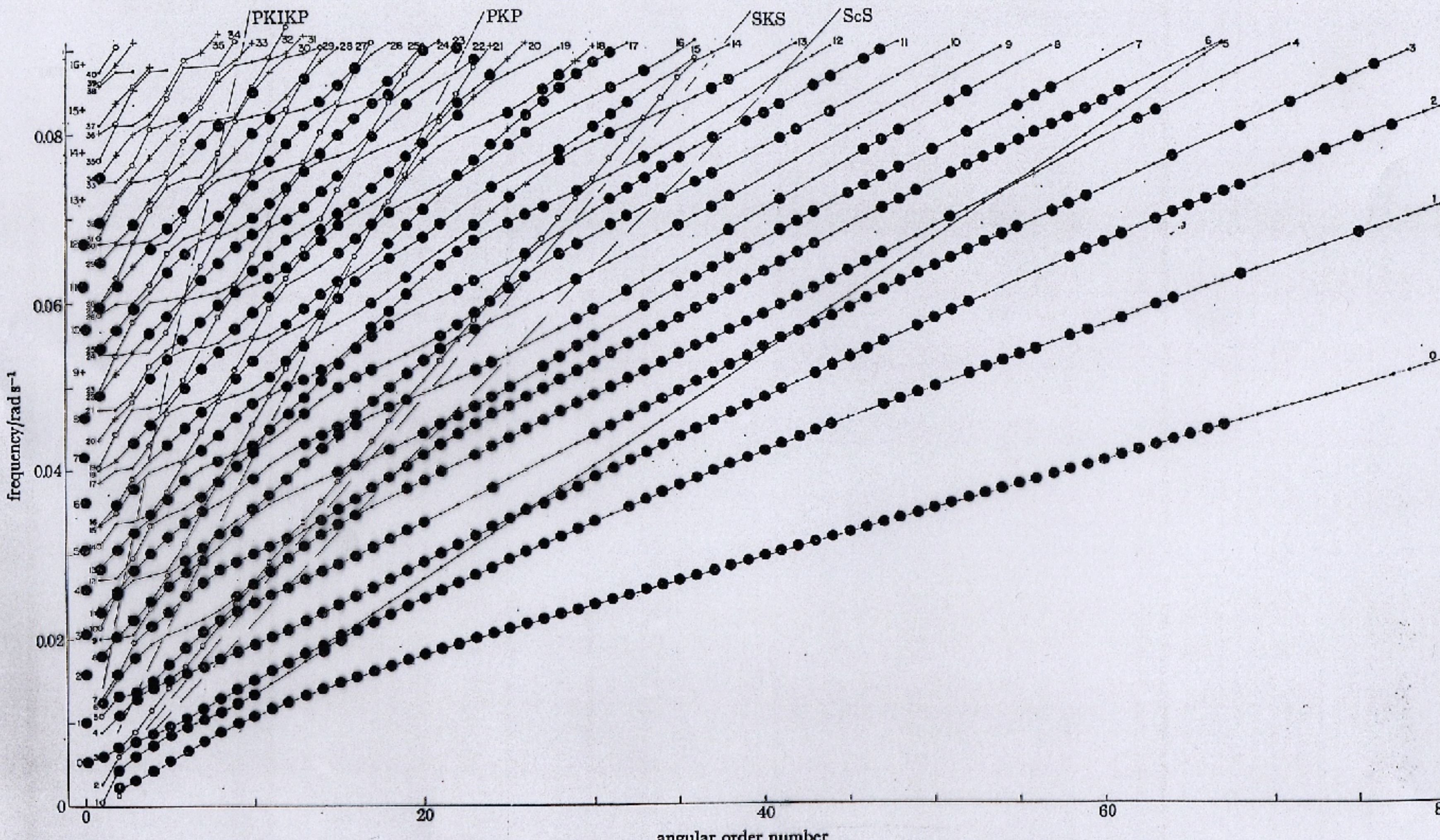


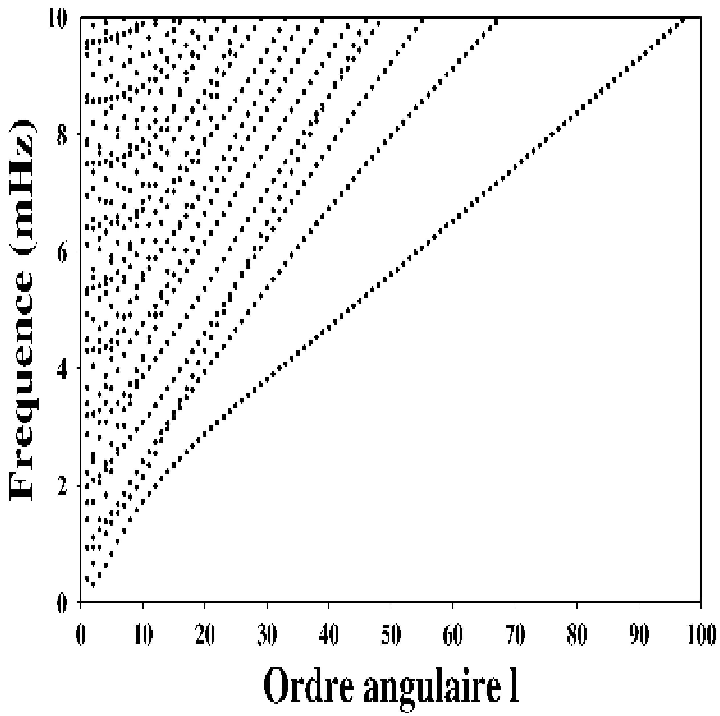
FIGURE 17. Spheroidal normal modes in the (ω, l) plane. The large dots indicate observed modes used in the inversions. For further details we refer the reader to §3 of Alaska II. •, $CE < 0.5$; +, $CE \geqslant 0.5$; ∘ core modes.

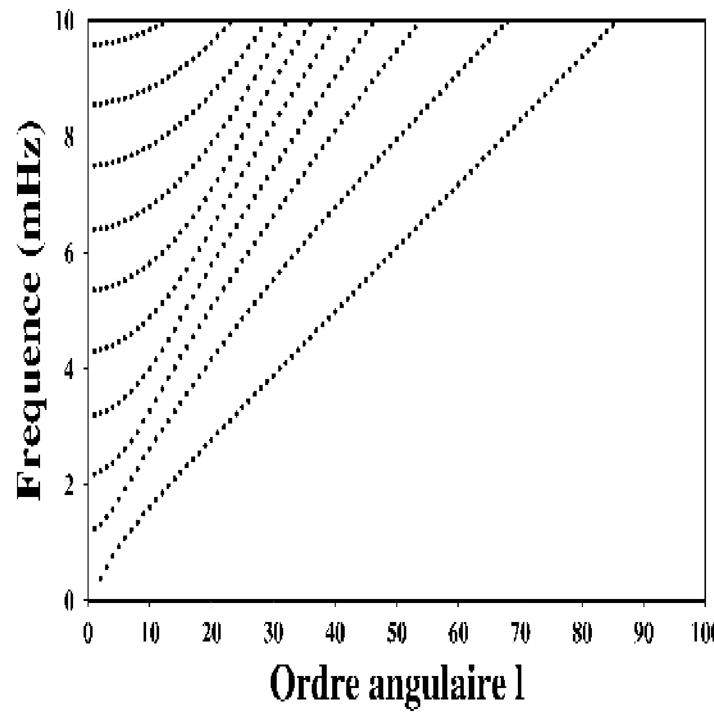
Spherical eigenfrequencies

Spheroïdal Modes $_{n}S_{l}$ Toroïdal modes $_{n}T_{l}$

(P-SV / Rayleigh) (SH / Love)

Dispersion Branches

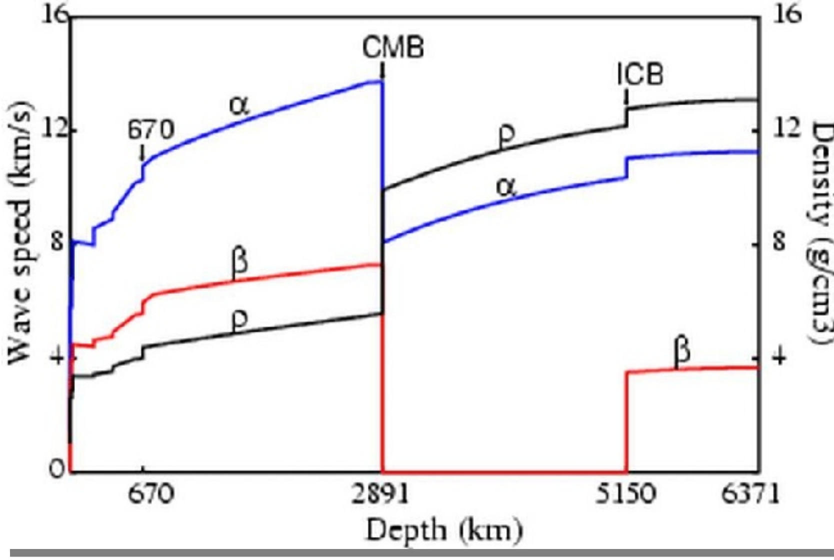




multiplet : (n,l) = 2l+1 singlets
singlet : (n,l,m)

n : radial order
l : angular order
m : azimuthal order

1D-Reference Earth Model:
$M_0(r)$, $\rho(r)$, $V_P(r)$, $V_s(r)$
(PREM, Dziewonski and Anderson, 1981)



$\rho\partial_{tt}\mathbf{u}_0 + \mathbf{H_0 u_0 = 0}$

Eigenfrequencies: ${}_n\omega_l$
Eigenfunctions: ${}_n u_l^m\,(r,t) = |n,l,m\rangle$

2 kinds of mode: Toroïdal ${}_nT_l$, Spheroïdal ${}_nS_l$

Degeneracy of eigenfrequencies ${}_n\omega_l$: 2 l +1

$\rho\partial_{tt}\mathbf{u}_0 = \mathbf{H_0 u_0}$ **(+ Fs)**

Eigenfrequencies: ${}_n\omega_l$

Eigenfunctions: ${}_nu_l^m\ (r,t) = |n,l,m>$

3 quantum numbers (k={n,l,m}) => $\mathbf{u}_k(r,t)$

$\int \rho\ \mathbf{u}_k^* \cdot \mathbf{u}_k\ d^3x = \delta_{ij}$

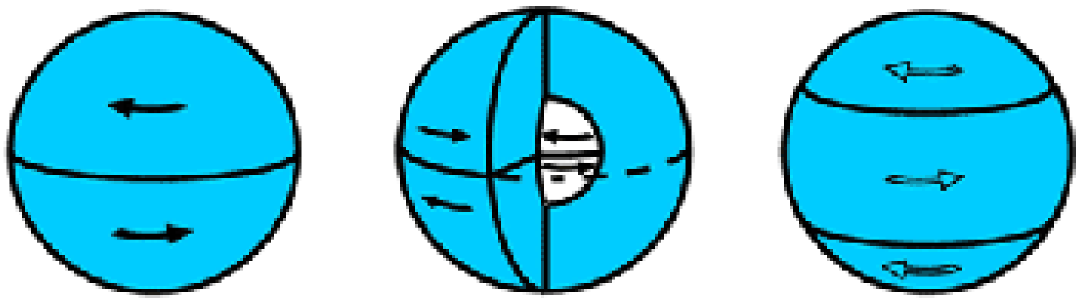
$\mathbf{H_0}\, \mathbf{u}_k = \rho\ {}_n\omega_l^2\, \mathbf{u}_k$

Displacement:

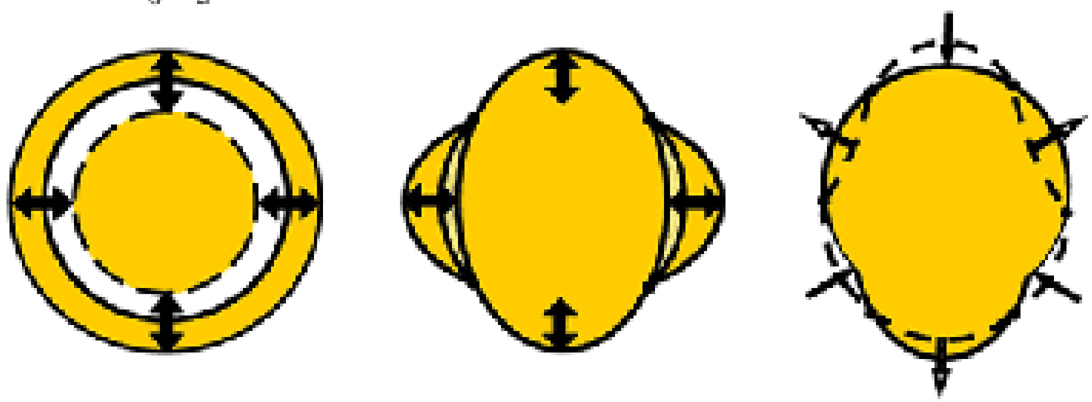
$\mathbf{u}(\mathbf{r},t) = \Sigma_{n,l,m}\ {}_n\mathbf{a}_l^m\ |n,l,m>\ \exp(-i\, {}_n\omega_l t)$

$\mathbf{u}_k(r,t) = \{U(r)\, \mathbf{e}_r + V(r)\mathbf{e}_\theta\, \partial_\theta + V(r)/\sin\theta\ \mathbf{e}_\phi\, \partial_\phi\}\ Y_l^m(\theta,\phi)$

$+ \{W(r)\, \mathbf{e}\, \partial - W(r)\, \mathbf{e}\, \partial\}\ Y^m(\theta,\phi)$



Toroidal modes $_0T_2$ (44.2 min), $_1T_2$ (12.6 min) and $_0T_3$ (28.4 min)

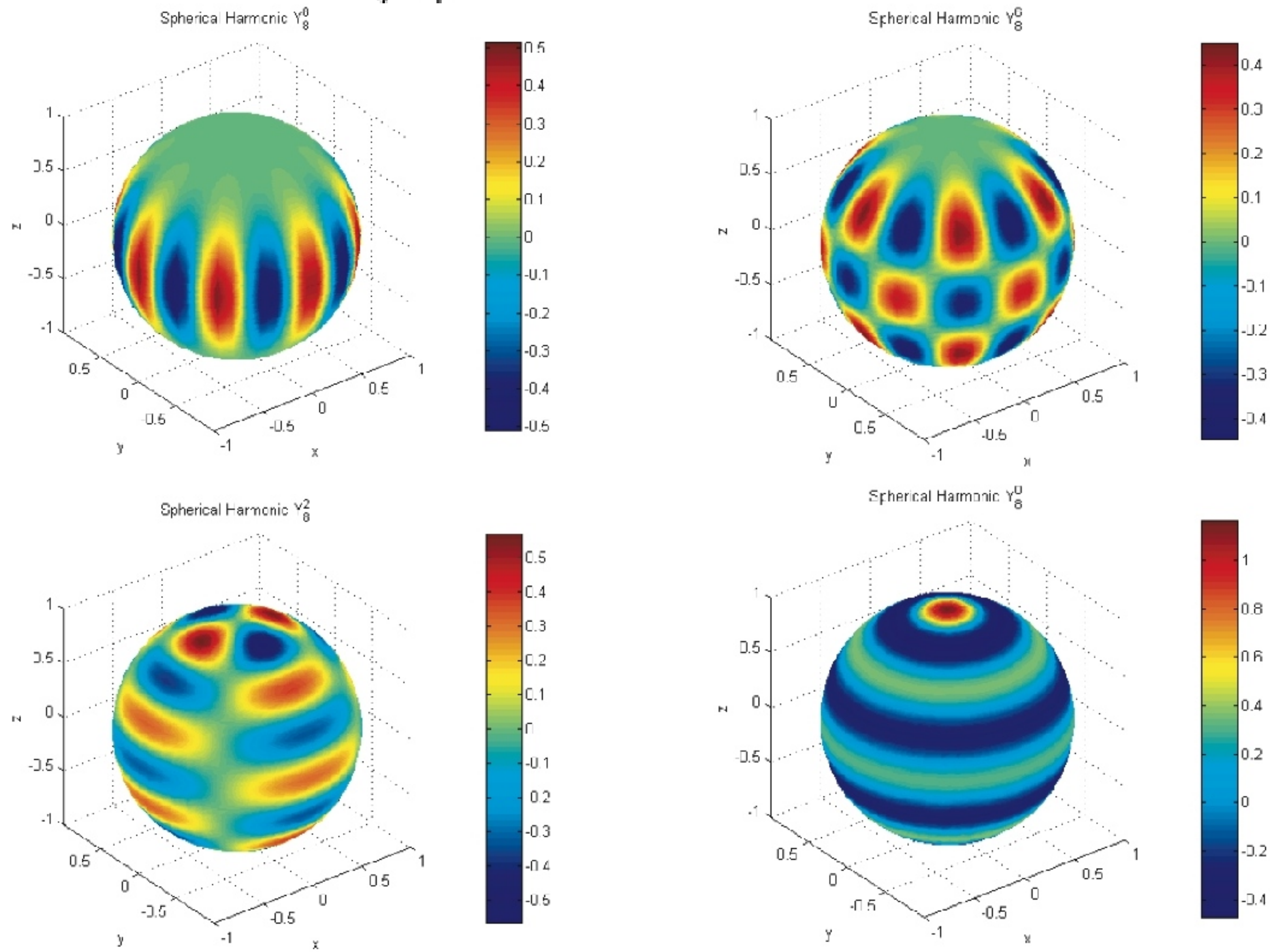


Spheroidal modes $_0S_0$ (20.5 min), $_0S_2$ (53.9 min) and $_0S_3$((35.6 min)

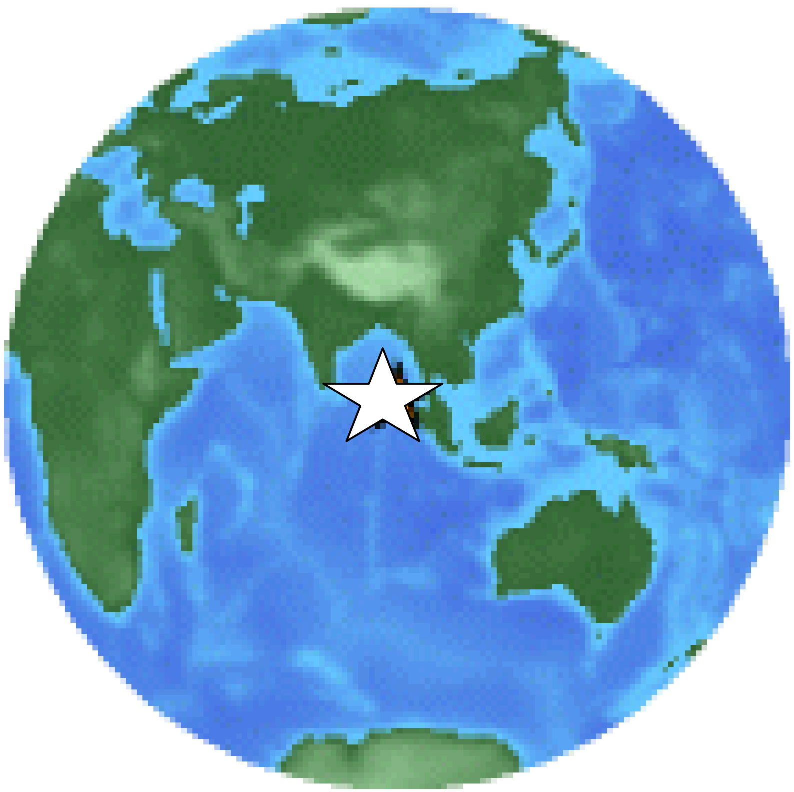
Parameterization, or choice of basis functions

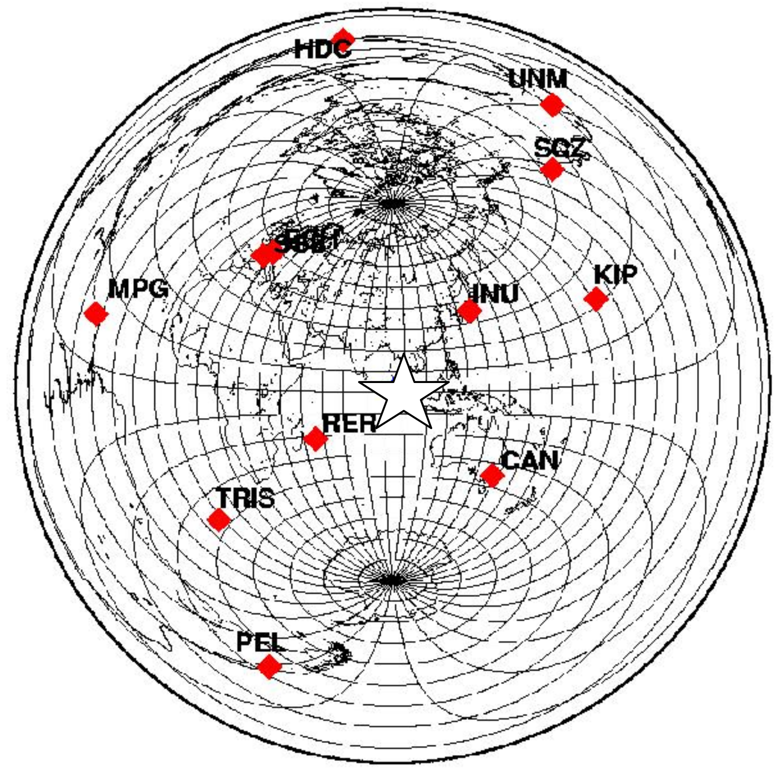
$$\delta v(\mathbf{r}) = \sum_{i=1}^{N} c_i f_i(\mathbf{r})$$

spherical harmonics ("global" basis functions)



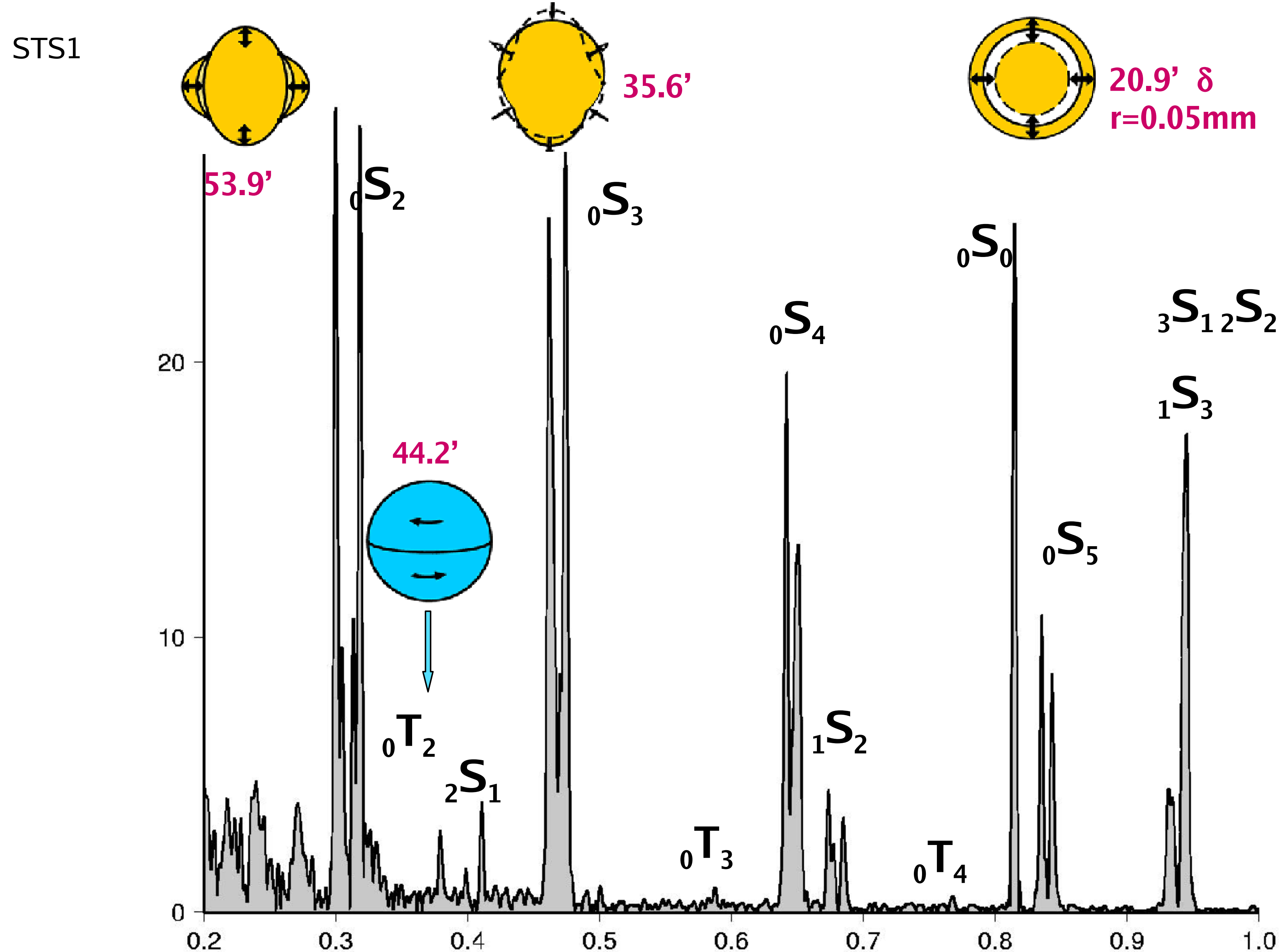
GEOSCOPE and Sumatra-Andaman earthquake (26 dec. 2004)





Study of Sumatra earthquake (26 december 2004)

(Roult and Clévédé, 2005 ; Park et al., Science, 2005)



Sumatra-Andaman earthquake
26 December 2004

Roult and Clévédé, 2005

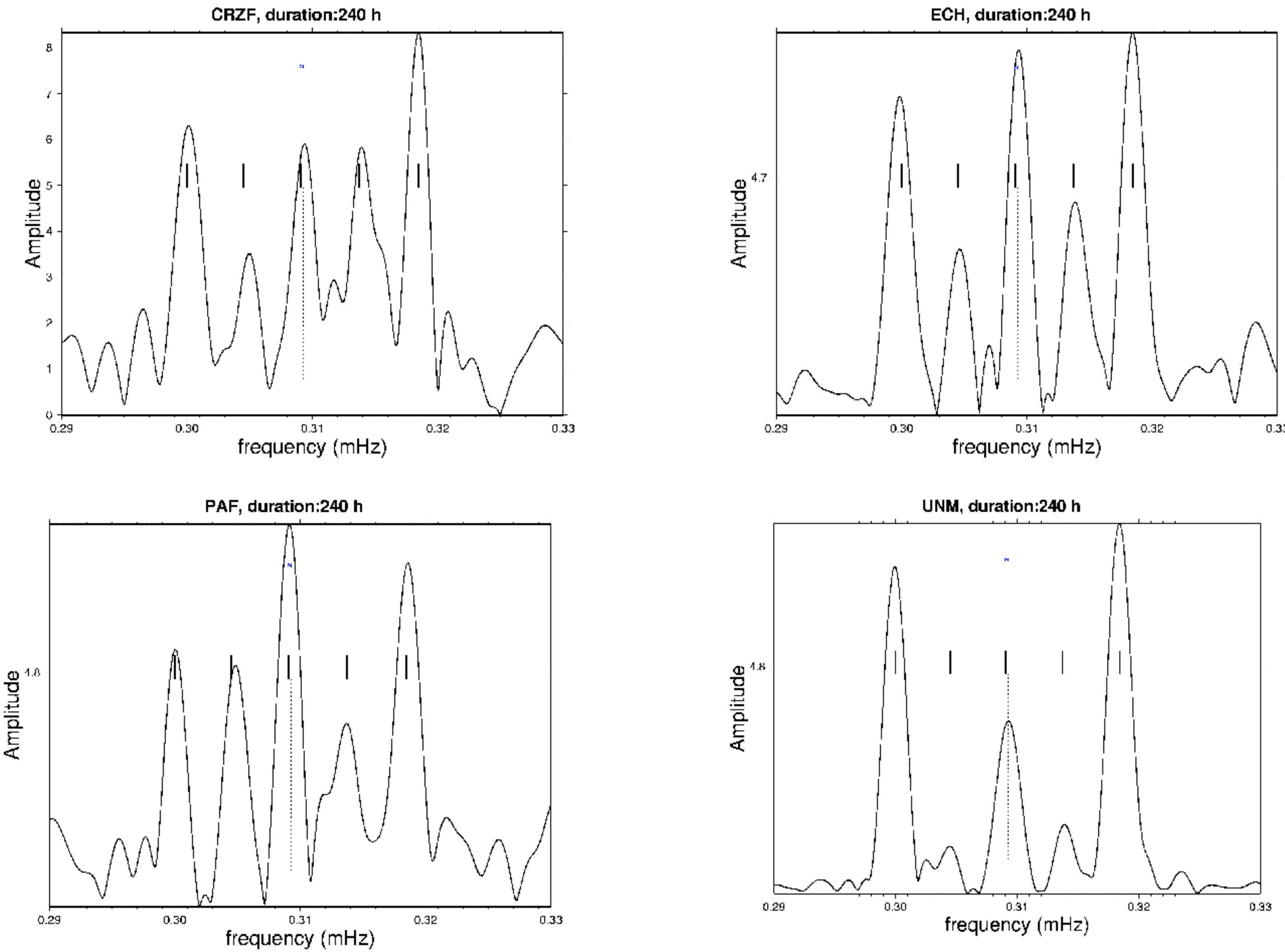
Effect of rotation on normal modes

$$\rho_0 \partial_{tt} \mathbf{u} + 2\rho_0 \mathbf{\Omega} X \partial_t \mathbf{u} = H \mathbf{u}(\mathbf{r}, t) + \mathbf{f}(\mathbf{r}, t)$$

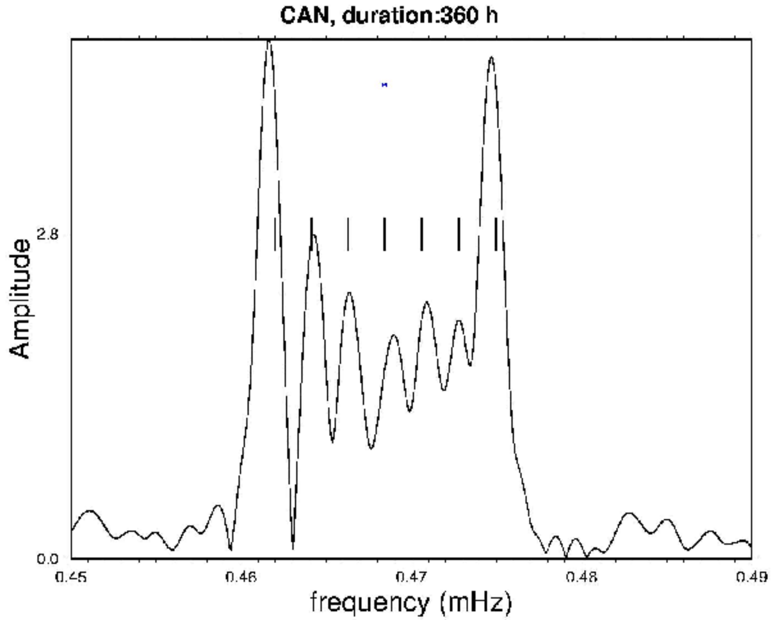
$${}_n\omega_\ell^m = {}_n\omega_\ell - \frac{m}{\ell(\ell+1)} \Omega \, \beta_n^\ell$$

β_n^ℓ Factor different according to the type of mode (spheroidal or toroidal)

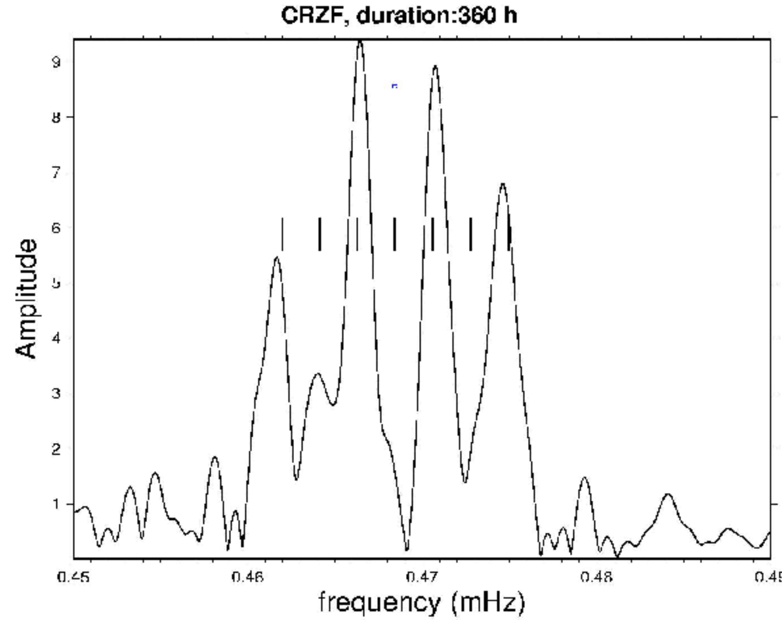
mode ${}_0S_2$ => splitting 5 singlets



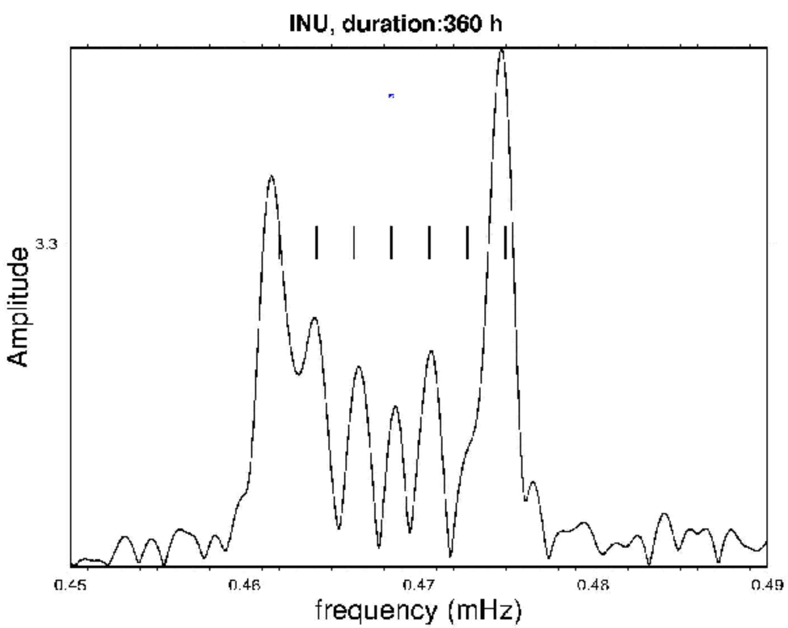
mode ${}_0S_3$



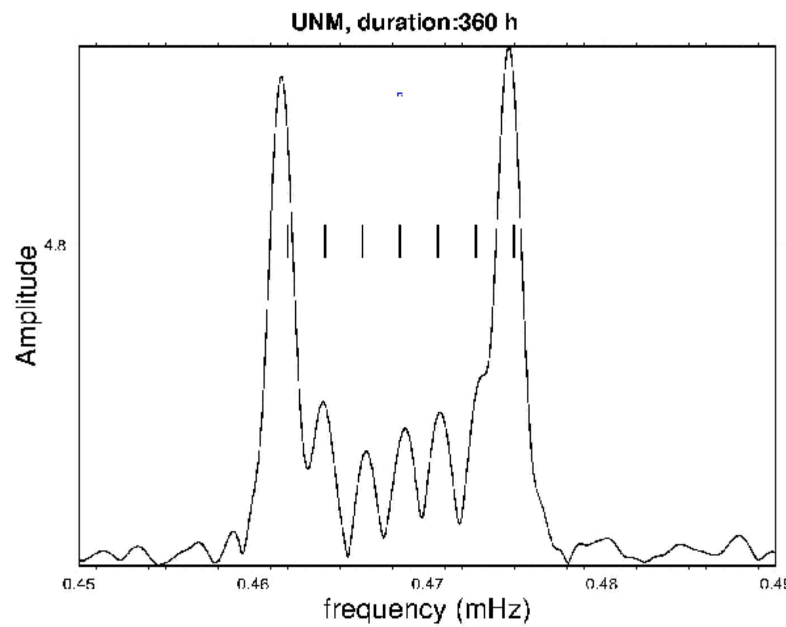
CAN, duration:360 h
Amplitude
2.8
0.0
0.45
0.46
0.47
0.48
0.49
frequency (mHz)



CRZF, duration:360 h
Amplitude
9
8
7
6
5
4
3
2
1
0.45
0.46
0.47
0.48
0.49
frequency (mHz)



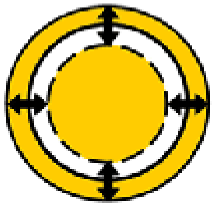
INU, duration:360 h
Amplitude
3.3
0.45
0.46
0.47
0.48
0.49
frequency (mHz)



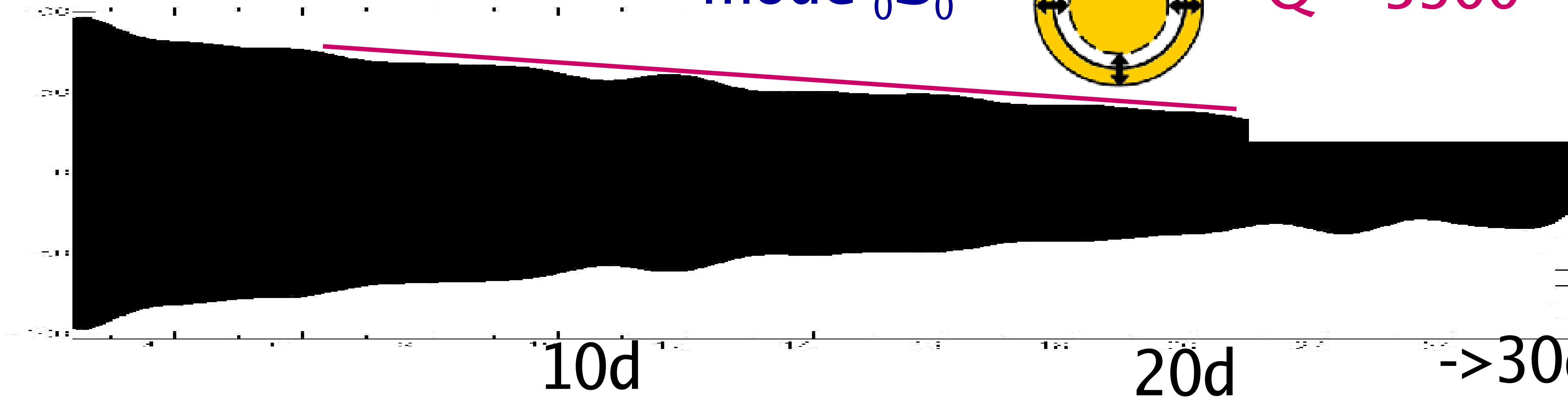
UNM, duration:360 h
Amplitude
4.8
0.45
0.46
0.47
0.48
0.49
frequency (mHz)

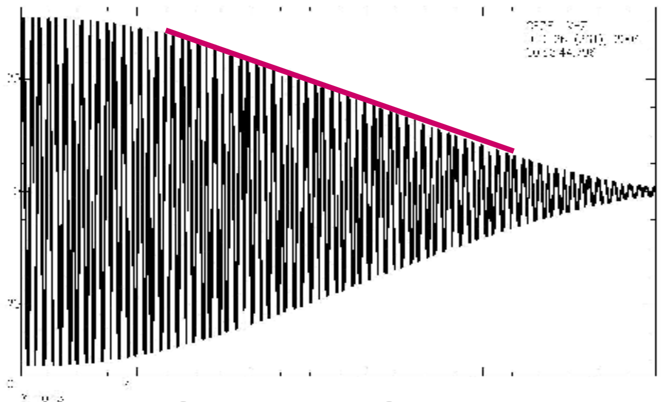
Attenuation of some modes

mode ${}_0S_0$

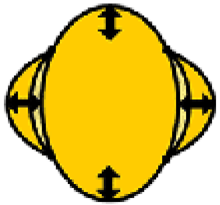


Q ~ 5300





mode ${}_0S_2$
singlet m=+2



Q ~ 500

CRZF, Crozet, Indian Ocean

Seismic Source

$$\rho\partial_{tt}\mathbf{u} + \mathbf{H_0 u} = \mathbf{F_s}$$

Displacement in point **r** at time t due to a force system $\mathbf{F_s}$ at point source $\mathbf{r_s}$

eigenfrequencies: ${}_n\omega_l$

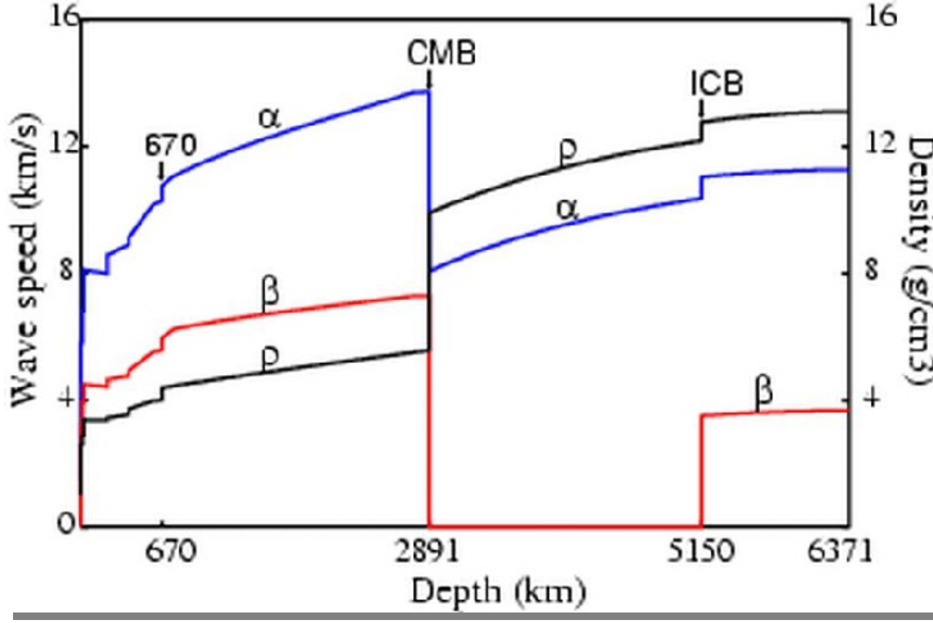
eigenfunctions: ${}_n u_l^m$ (r,t)= |n,l,m>

$$\mathbf{u}(\mathbf{r},t) = \Sigma_{n,l,m}\ {}_n\mathbf{a}_l^m\ |n,l,m\rangle\ \exp(-i\,{}_n\omega_l t)$$

Eigenfunction basis is a complete basis => any wave can be modelled by normal mode summation including surface waves and body waves.

1D- Reference Earth Model

- Normal mode calculation (**eigenfrequencies** $_n\omega_k$ **eigenfunctions** $_nu_l(r,\mathbf{t})$)
- Synthetic Seismograms by normal mode summation ($k=\{n,l,m\}$).

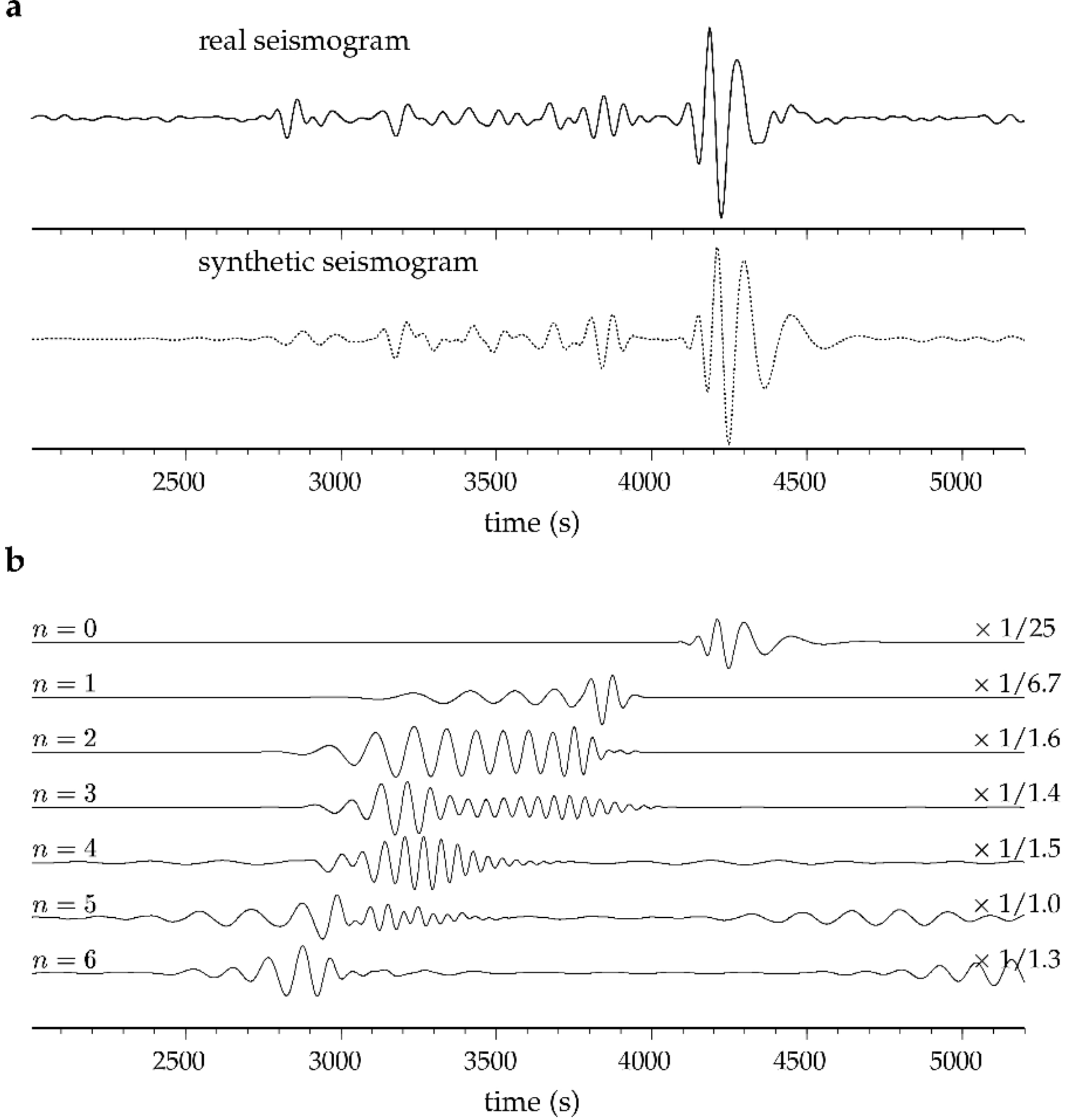


$\mathbf{u}(\mathbf{r},t)$ Displacement at point **r** at time t due to a force system **F** at point source $\mathbf{r}_s$

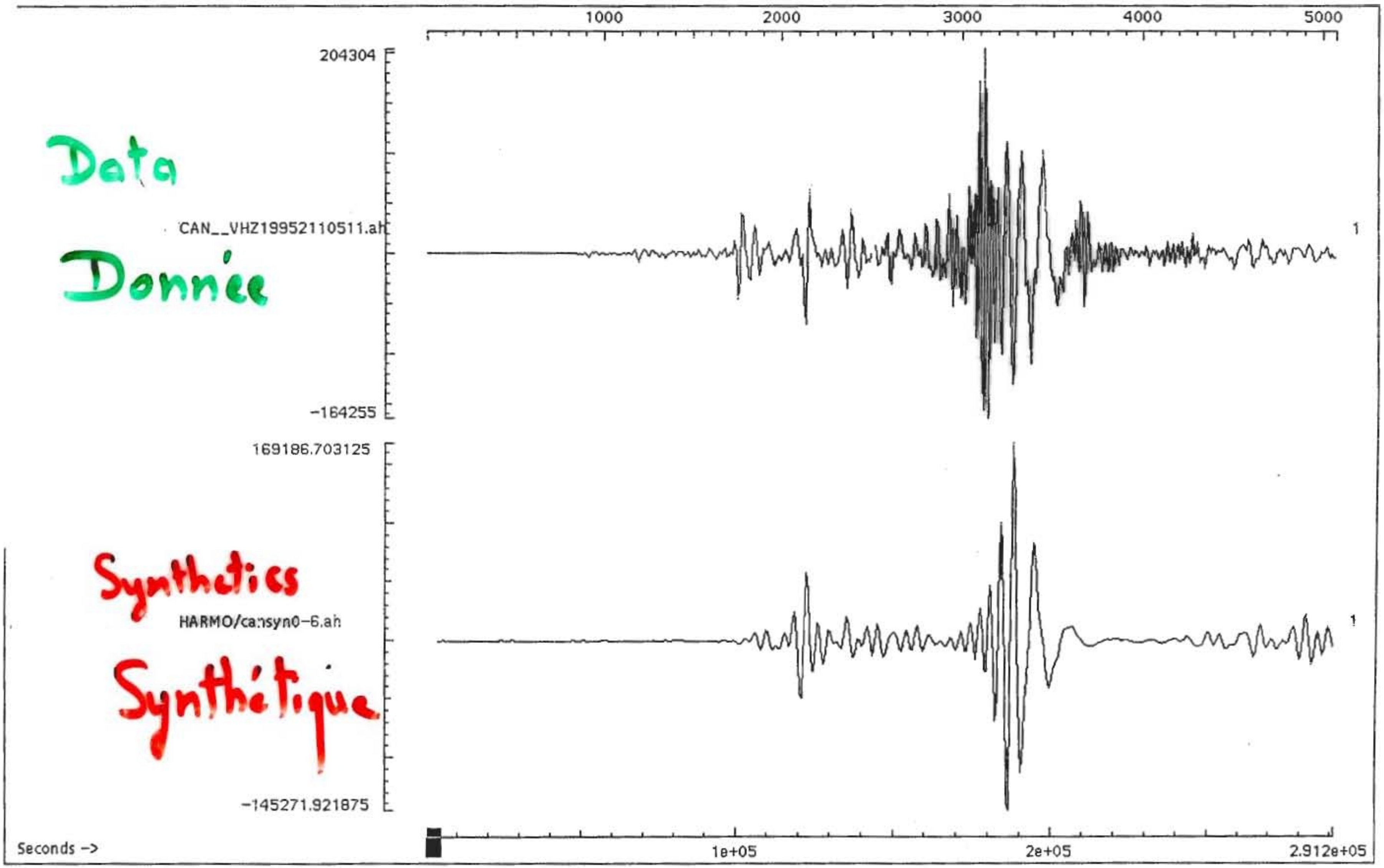
$$\mathbf{u}(\mathbf{r},t) = \Sigma_k\, u_k(r) \cos \omega_k t / \omega_k^2 \exp(-\omega_k t/2Q)\, (\mathbf{u_k}.\mathbf{F})_\mathbf{S}$$

Source Term $(\mathbf{u_k}.\mathbf{F})_\mathbf{S} = (\mathbf{M}:\boldsymbol{\varepsilon})_S$

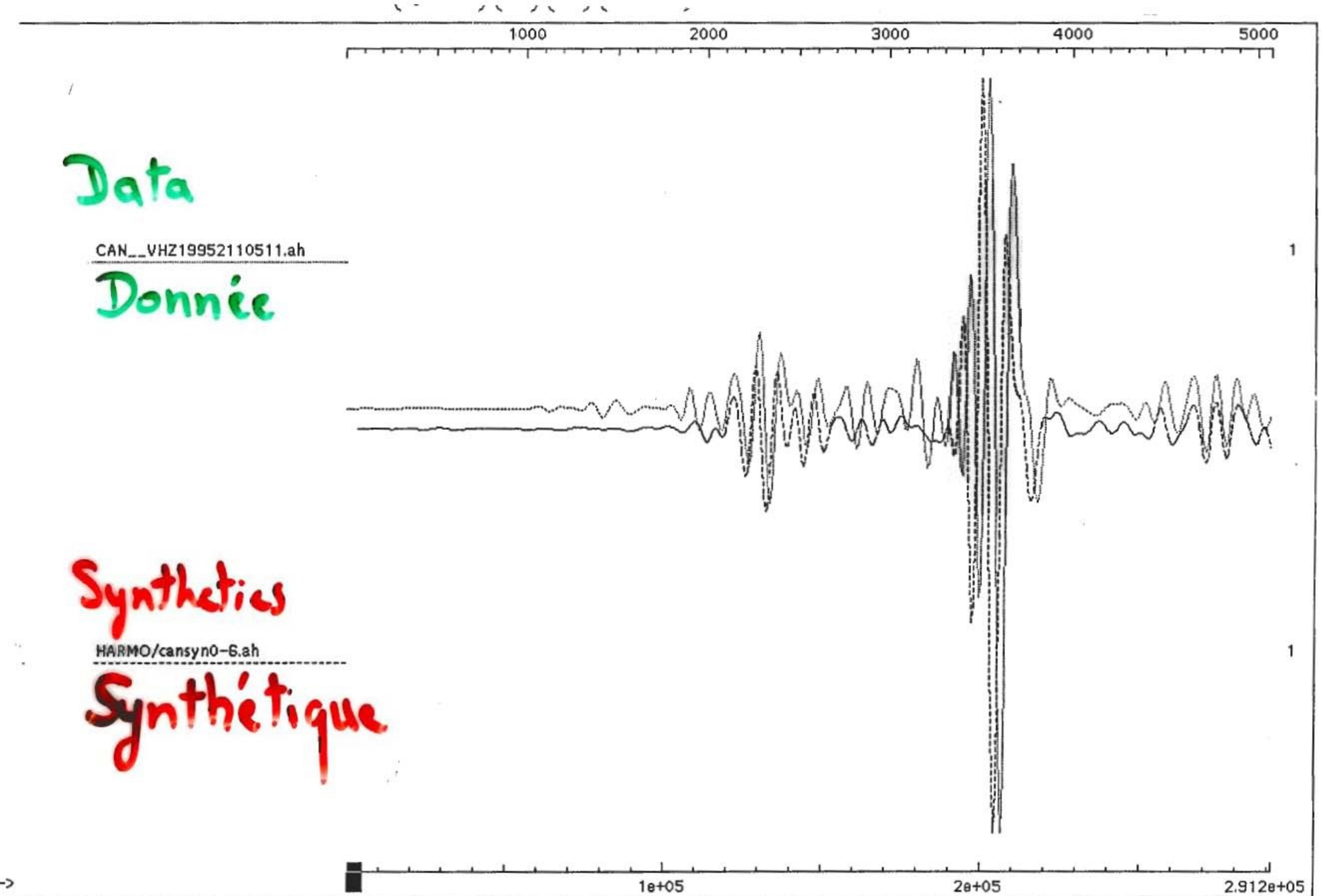
M Seismic moment tensor, **ε** deformation tensor



Beucler et al., 2003



1000
2000
3000
4000
5000
204304
Data
CAN__VHZ19952110511.ah
Donnée
-164255
1
169186.703125
Synthetics
HARMO/cansyn0-6.ah
Synthétique
-145271.921875
1
Seconds ->
1e+05
2e+05
2.912e+05


1000
2000
3000
4000
5000
Data
CAN__VHZ19952110511.ah
Donnée
1
Synthetics
HARMO/cansyn0-6.ah
Synthétique
1
1e+05
2e+05
2.912e+05

Eigenfunction basis is a complete basis => any wave can be modelled by normal mode summation including surface waves and body waves.

Asymptotic form of $Y_l^m(\theta,\phi)$:

$Y_l^m(\theta,\phi) := \pi^{-1} (\sin\theta)^{1/2} \cos[(l+1/2)\theta + 1/2\, m\pi - 1/4\, \pi]\, e^{im\phi}$

For a source at the pole, θ plays the role of epicentral distance.

The horizontal wavenumber k is: $k = (l + {}^1/_2)/a$

And phase velocity is $c(\omega) = {}^{\omega}/_{k}$

Ray parameter $p = a \sin i / {}_{V}$ <=> horizontal slowness

$^{k}/$ $(l + 1/2)$

From normal modes to surface waves

Asymptotic form of $Y_l^m(\theta,\phi)$:

$Y_l^m(\theta,\phi) := \pi^{-1} (\sin\theta)^{1/2}\cos[(l+1/2)\theta + 1/2\, m\pi - 1/4\, \pi]\, e^{im\phi}$

For a source at the pole, θ plays the role of epicentral distance.

The horizontal wavenumber k is: $k=(l+{}^1/_2)/a$

And phase velocity is $c(\omega)= {}^{\omega}/_{k}$

$$\sum_{l=0}^{\infty} f(l+{}^1/_2) = \int_{-\infty}^{+\infty} f(\nu)\, e^{-i\pi\nu} \sum_{q=0}^{\infty} e^{-2i\pi\nu q}\, d\nu$$

Sum on modes replaced by sum on trains

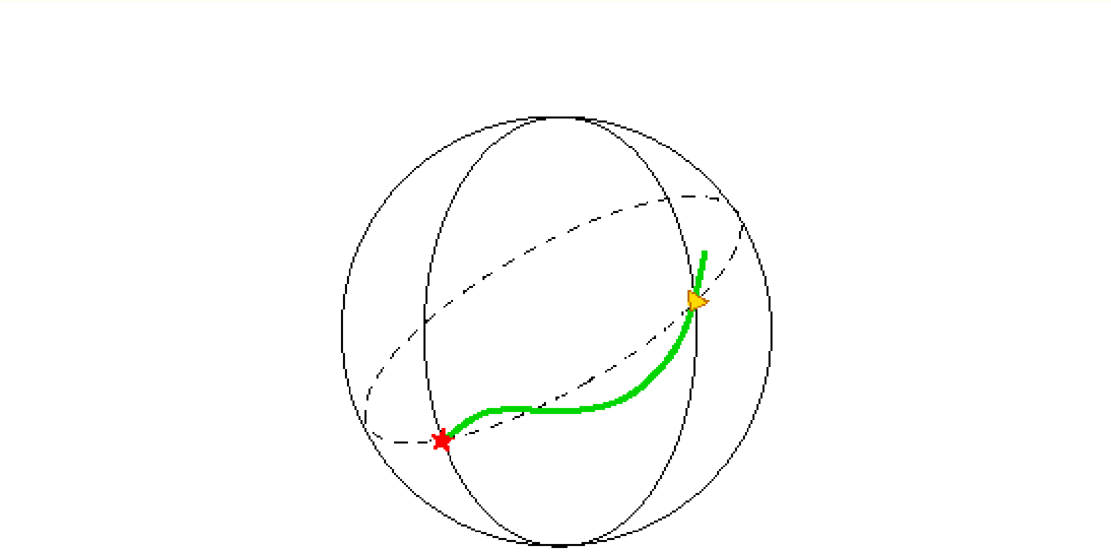
$$U_r(r,\theta,\phi) = \sum_{n} \sum_{q=0}^{\infty} \int_{-\infty}^{+\infty} A(\kappa)\, (e^{i\omega(t-\Delta/c)+\phi+} + e^{i\omega(t+\Delta/c)+\phi-})\, d\kappa$$

Surface wave full ray theory

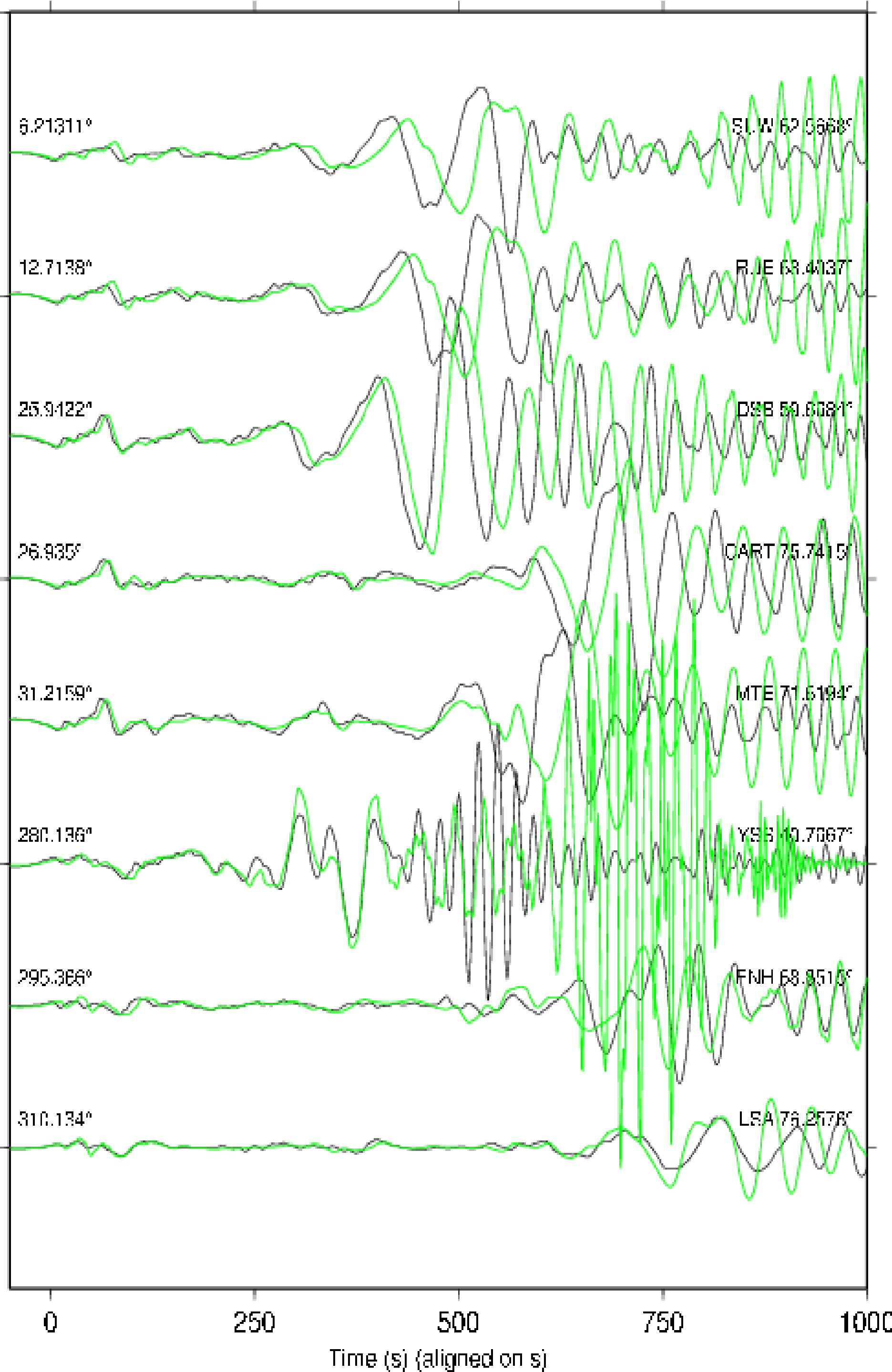
Waves locally as plane waves $\lambda \ll \Lambda_\Omega$

$$u(\omega) = \sum_{branches} \sum_{orbits} A(\omega) \exp^{-i\phi(\omega)}$$

$$A = A_s A_p A_r \qquad \phi = \phi_s + \phi_p + \phi_r$$



Woodhouse 1974, Woodhouse & Wong 1986, Tromp & Dahlen 1992, Ferreira & Woodhouse 2007a.



Synthetic seismograms By normal mode summation

Denali-Alaska earthquake (Nov. 2002)

Komatitsch and Tromp, 2003

Duality wave - particle:
λ seismic wavelength
Λ scale heterogeneity

Particle: Ray theory $\lambda \ll \Lambda$

Wave: Normal mode theory (NM) + Perturbation theories (small amplitude of 3D-heterogeneities) -> Global tomography

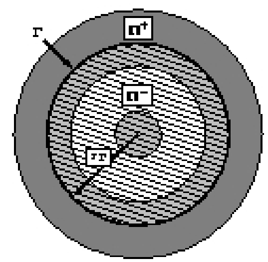
Numerical modelling of wave equation
Strong or weak forms: $\lambda \approx \Lambda$
-Spectral Element Method (SEM)
-Coupled SEM-NM method

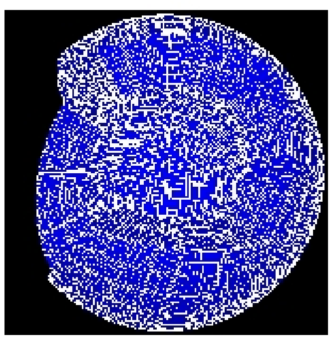
Spectral Element Method: D. Komatitsch (1999)

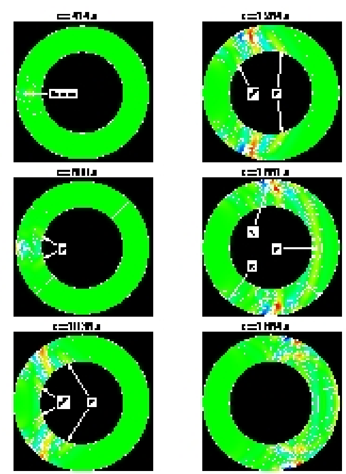
Coupled method of Spectral Elements and Modal Solution

Principle:

- Ω^+: Spectral Element area: 3D model
- Ω^-: Modal Solution area: 1D model







Capdeville et al., 2002

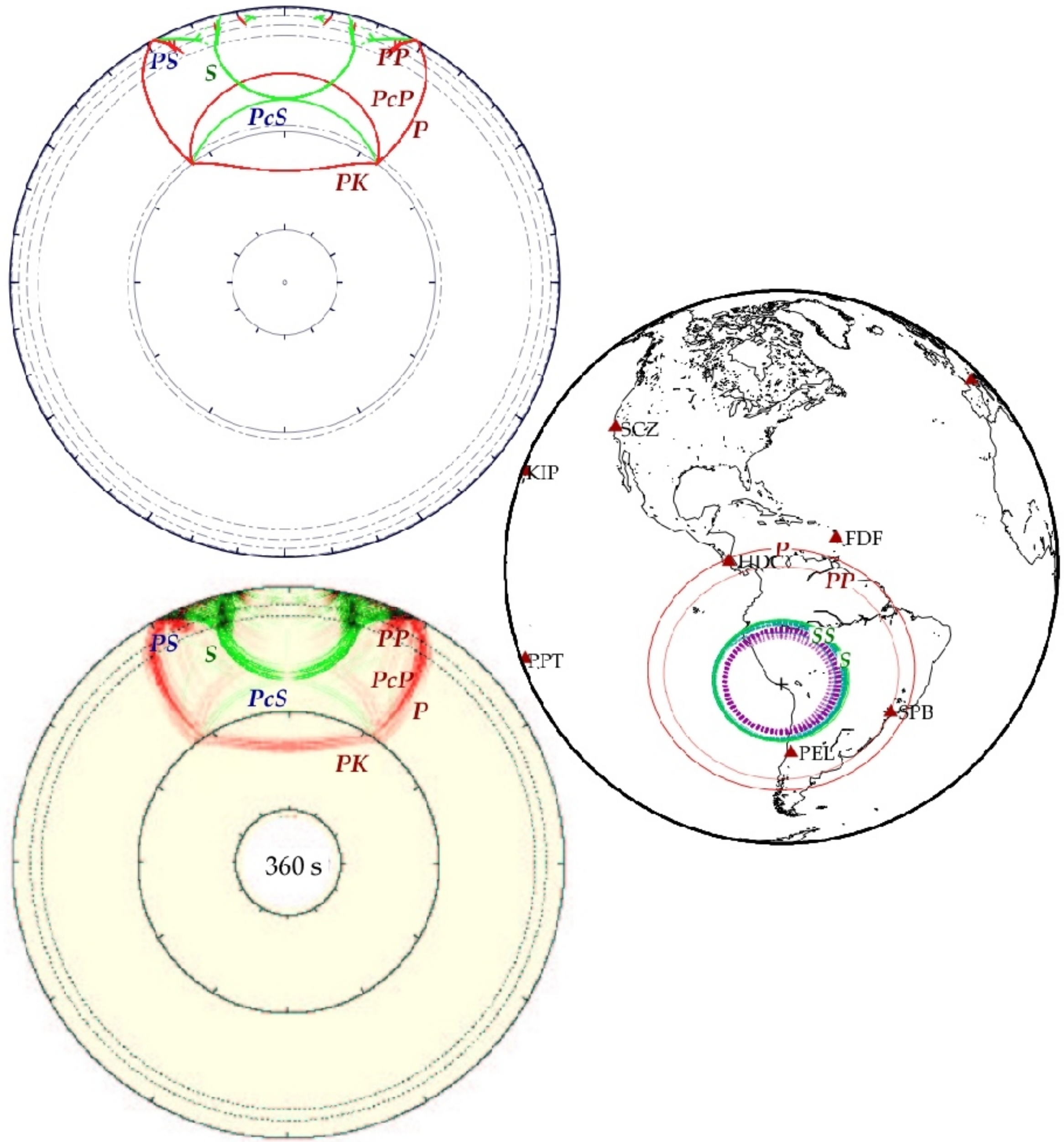


Figure 1.6. Wavefronts for the Peruvian event at 360 s after source initiation, together with a snapshot of the wavefield from a numerical simulation.

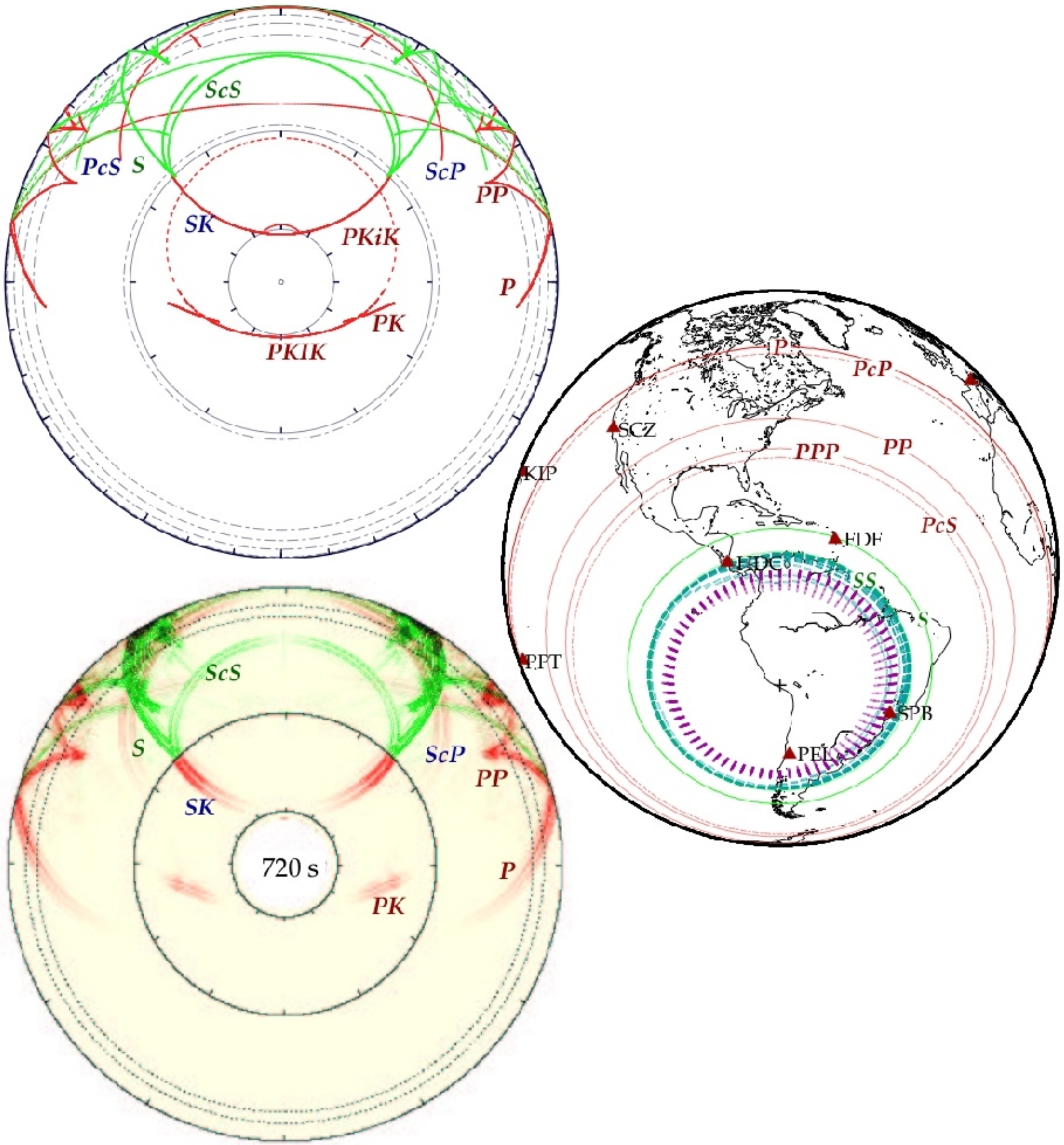


Figure 1.8. Wavefronts for the Peruvian event at 720 s after source initiation, together with a snapshot of the wavefield from a numerical simulation.

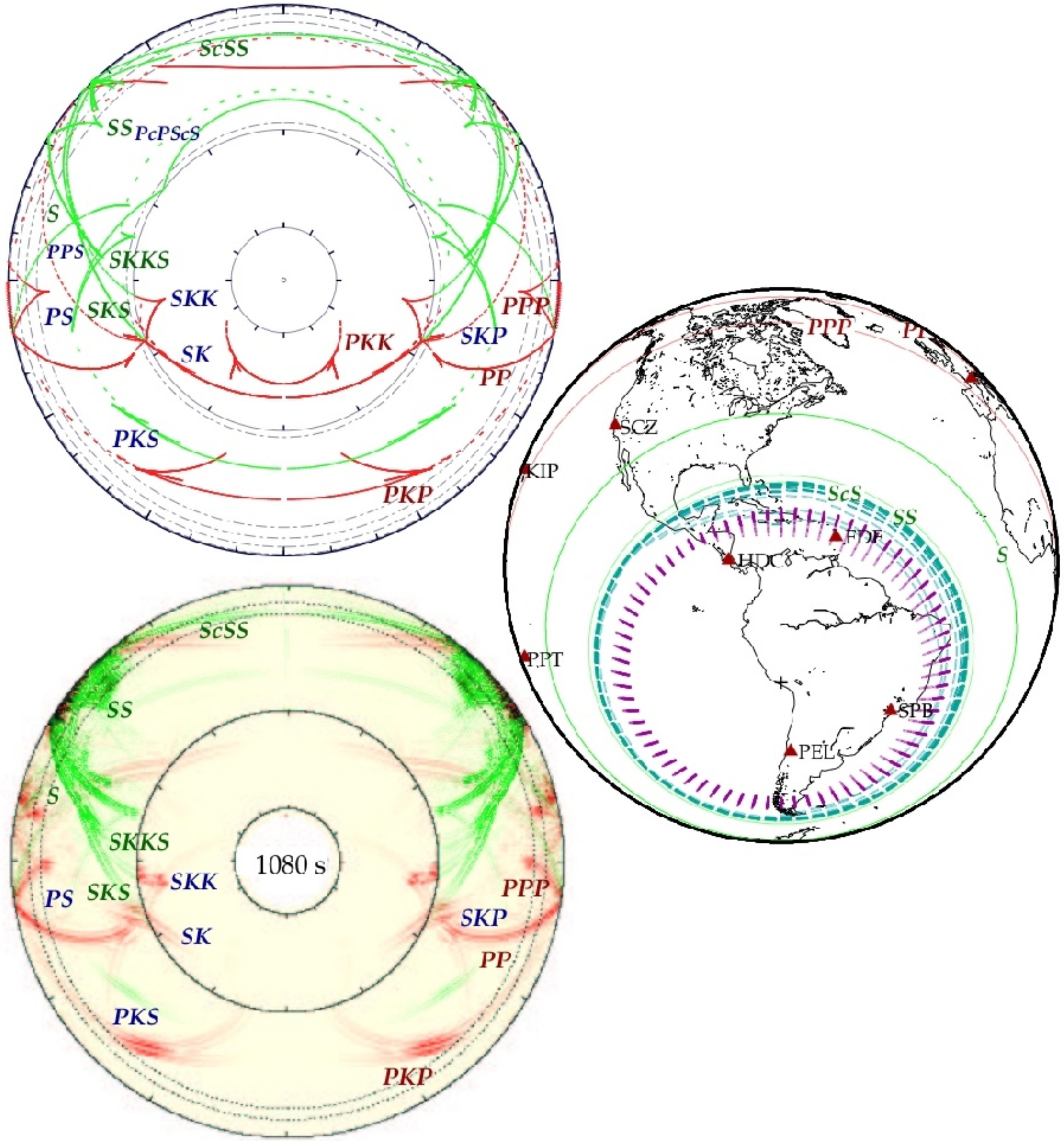


Figure 1.10. Wavefronts for the Peruvian event at 1080 s after source initiation, together with a snapshot of the wavefield from a numerical simulation.

Overview
Large scale Seismology: an observational field

- Data (Seismic source) + Instrument (Seismometer) -> Observations (seismograms)
- Historical evolution: Ray theory, Normal mode theory, Numerical techniques (SEM, NM-SEM)

- Scientific Issues: earthquakes, structure of the Earth and planets

- Seismic Experiment: Plume dectection
- NM-SEM and time reversal

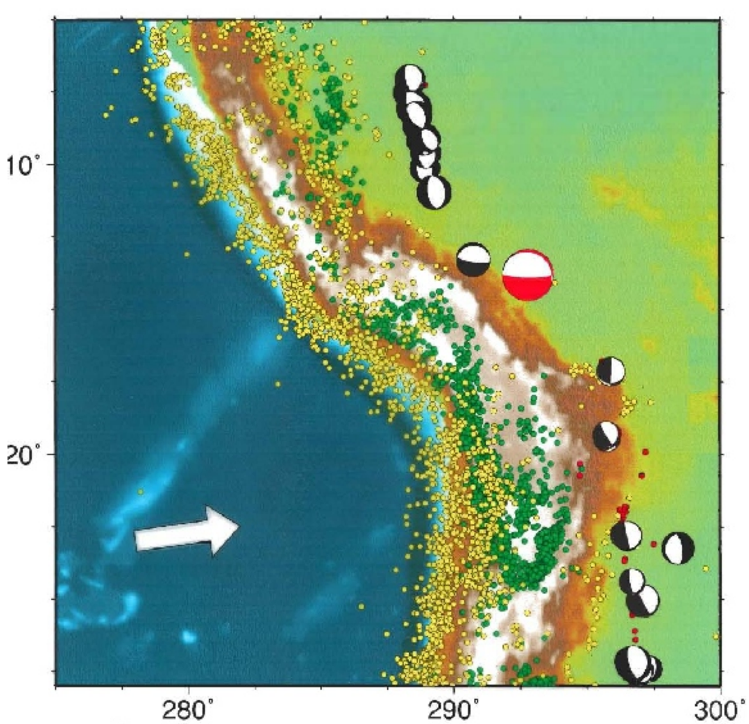
Seismic Source Studies

$$\mathbf{u}(\mathbf{r},t) = \Sigma_k\, u_k\,(\,r\,) \cos \omega_k t\,/\omega_k^{\,2} \exp(-\omega_k t/2Q)\,(\mathbf{u_k}.\mathbf{F})_S$$

Source Term $(\mathbf{u_k}\,.\mathbf{F})_S = (\mathbf{M}:\boldsymbol{\varepsilon})_S$

M Seismic moment tensor, **ε** deformation

Bolivia 94/06/09 $M_w = 8.2$



CMT depth > 350 km 1977-1994

NEIC magn > 4.5 1962-1994