TD Thermo 1

1 – $f(P,V,T)=0. \Rightarrow df = \frac{\partial f}{\partial P}dP + \frac{\partial f}{\partial V}dV + \frac{\partial f}{\partial T}dT = 0.$

a. $P = cste \quad \frac{\partial f}{\partial V}dV + \frac{\partial f}{\partial T}dT = 0 \Rightarrow \left.\frac{\partial T}{\partial V}\right)_{P\,const} = \frac{-\partial f/\partial V}{\partial f/\partial T}$

$V = cste \quad \frac{\partial f}{\partial P}dP + \frac{\partial f}{\partial T}dT = 0 \Rightarrow \left.\frac{\partial P}{\partial T}\right)_{V\,cst} = \frac{-\partial f/\partial T}{\partial f/\partial P}$

$T = cst \quad \frac{\partial f}{\partial P}dP + \frac{\partial f}{\partial V}dV = 0 \Rightarrow \left.\frac{\partial V}{\partial P}\right)_{T\,cst} = \frac{-\partial f/\partial P}{\partial f/\partial V}$

donc $\left.\frac{\partial T}{\partial V}\right)_P \cdot \left.\frac{\partial P}{\partial T}\right)_V \cdot \left.\frac{\partial V}{\partial P}\right)_T = \frac{-\cancel{\partial f/\partial V}}{\cancel{\partial f/\partial T}} \cdot \frac{-\cancel{\partial f/\partial T}}{\partial f/\partial P} \cdot \frac{-\cancel{\partial f/\partial P}}{\cancel{\partial f/\partial V}} = -1.$

b. $\alpha = \frac{1}{V}\cdot\left.\frac{\partial V}{\partial T}\right)_P \qquad \beta = \frac{1}{P}\left.\frac{\partial P}{\partial T}\right)_V \qquad \chi = -\frac{1}{V}\cdot\left.\frac{\partial V}{\partial P}\right)_T$

coef. dilatation isobare. coef. augmentation de P à V const coef. compressibilité isotherm.

$\beta\chi = -\frac{1}{PV}\cdot\left.\frac{\partial P}{\partial T}\right)_V \left.\frac{\partial V}{\partial P}\right)_T = -\frac{1}{PV}\cdot\frac{-\partial f/\partial T}{\partial f/\partial P}\cdot\frac{-\partial f/\partial P}{\partial f/\partial V} = -\frac{1}{PV}\cdot\frac{\partial f/\partial T}{\partial f/\partial V}$

et $\alpha = \frac{1}{V}\cdot\left.\frac{\partial V}{\partial T}\right)_P = \frac{1}{V}\cdot\frac{-\partial f/\partial T}{\partial f/\partial V} \rightsquigarrow \beta\chi = \frac{1}{P}\cdot\alpha \Rightarrow \boxed{\alpha = \beta\chi P.}$

2 – $\alpha = \frac{nR}{PV}$; $\beta = \frac{1}{T}$

équation d'état = 3 variable (P,V,T), chaque coefficient thermoélastiques = 2 variable

$\Rightarrow$ 2 coef. nécessaire !

<u>P = cste</u> : $\alpha = \frac{1}{V}\left.\frac{\partial V}{\partial T}\right)_P = \frac{nR}{PV} \rightarrow \frac{1}{V}\cdot dV = \frac{nR}{PV}\cdot dT \Rightarrow V = \frac{nRT}{P} + c(P) \quad \forall P!$

<u>V = cste</u> : $\beta = \frac{1}{P}\left.\frac{\partial P}{\partial T}\right)_V = \frac{1}{T} \rightarrow \frac{1}{P}\cdot dP = \frac{1}{T}dT$

<u>détermination de</u> $c(P)$: $\forall P \quad V = \frac{nRT}{P} + c(P)$ or à $V = cst$, $\frac{1}{P}dP = \frac{1}{T}dT$

et $dV = 0 = \left.\frac{\partial V}{\partial T}\right)_P dT + \left.\frac{\partial V}{\partial P}\right)_T \cdot dP$

$0 = \frac{nR}{P}dT - \frac{nRT}{P^2}dP + dc(P)$

or $\frac{dP}{P} = \frac{dT}{T} \rightarrow 0 = \frac{nR}{P}dT - \frac{nR}{P}dT + dc(P) = dc(P) \Rightarrow c(P) = K$

$\Rightarrow \forall P,V,T \quad V = \frac{nRT}{P} + K$ ou $PV = nRT + PK \quad \boxed{P(V-K) = nRT}$

(3)

3 (a) $dU = \delta Q + \delta W = \delta Q - P\,dV$

(b) $dU = \left.\frac{\partial U}{\partial T}\right)_V dT + \left.\frac{\partial U}{\partial V}\right)_T dV \rightarrow \delta Q = dU + P\,dV = \underbrace{\left.\frac{\partial U}{\partial T}\right)_V}_{c_V} . dT + \underbrace{\left[\left.\frac{\partial U}{\partial V}\right)_T + P\right]}_{\ell} . dV$

$$\underline{\delta Q = c_V\,dT + \ell\,dV}$$

(c) $H = U + PV \rightarrow dH = dU + P\,dV + V\,dP = \delta Q - P\,dV + P\,dV + V\,dP = \delta Q + V\,dP$

$$\boxed{\delta Q = dH - V\,dP}$$

or $dH = \left.\frac{\partial H}{\partial T}\right)_P dT + \left.\frac{\partial H}{\partial P}\right)_T dP \rightarrow \delta Q = \underbrace{\left.\frac{\partial H}{\partial T}\right)_P}_{c_P} . dT + \underbrace{\left[\left.\frac{\partial H}{\partial P}\right)_T - V\right]}_{h} dP$

$$\underline{\delta Q = c_P\,dT + h\,dP}$$

(d) $dU = \left.\frac{\partial U}{\partial P}\right)_V dP + \left.\frac{\partial U}{\partial V}\right)_P dV$

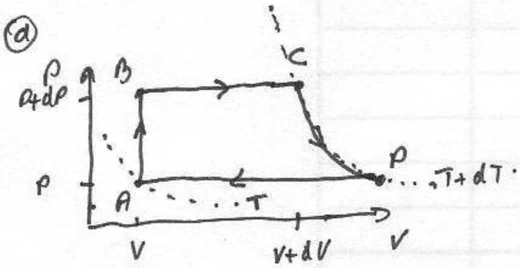
$\delta Q = dU + P\,dV = \underbrace{\left.\frac{\partial U}{\partial P}\right)_V}_{\lambda} dP + \underbrace{\left[\left.\frac{\partial U}{\partial V}\right)_P + P\right]}_{\mu} dV$

$$\underline{\delta Q = \lambda\,dP + \mu\,dV}$$

(e) U fonction d'état → OK

$\delta W = -P\,dV$ n'est pas vrai si irréversible → les expressions de δQ sont alors différentes !

4 — (a)



$dU = \phi = \delta Q + \delta W$

$\phi = \delta Q_{AB} + \delta Q_{BC} + \delta Q_{CD} + \delta Q_{DA} + \delta W$

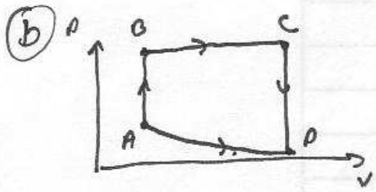
$\phi = (\lambda\,dP + \cancel{\mu\,dV})_{AB} + (\lambda\,\cancel{dP} + \mu\,dV)_{BC} + (c_P\,\cancel{dT} + h\,dP)_{CD} + (c_P\,dT + h\,\cancel{dP})_{DA} + \delta W$

$\phi = \lambda\,dP + \mu\,dV - h\,dP - c_P\,dT + \delta W$

$\delta W < 0 \simeq dP.dV \ll \delta Q_{ij}$

$\Rightarrow \lambda\,dP + \mu\,dV = h\,dP + c_P\,dT$

(b)



$\phi = \delta Q_{AB} + \delta Q_{BC} + \delta Q_{CD} + \delta Q_{DA} + \delta W$

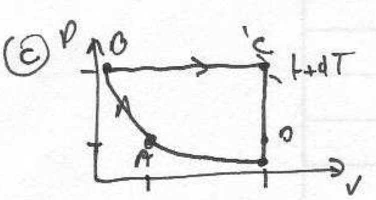
$= (\lambda\,dP + \mu\,\cancel{dV}) + (\lambda\,\cancel{dP} + \mu\,dV) + (-c_V\,dT + \ell\,\cancel{dV}) + (c_V\,\cancel{dT} - \ell\,dV) + \delta W \neq$

$\phi = \lambda\,dP + \mu\,dV - c_V\,dT - \ell\,dV + \delta W$

$< 0 \ll \delta Q_{ij}$.

$\Rightarrow \lambda\,dP + \mu\,dV = c_V\,dT + \ell\,dV$

(c)

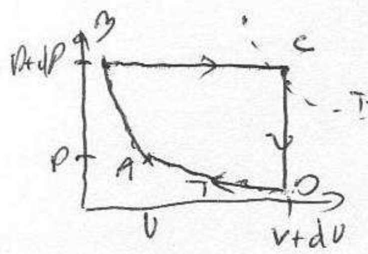


$\phi = \delta Q_{AB} + \delta Q_{BC} + \delta Q_{CD} + \delta Q_{DA} + \delta W$.

$= (c_P\,\cancel{dT} + h\,dP)_{AB} + (c_P\,dT + h\,\cancel{dP})_{BC} + (-c_V\,dT + \ell\,\cancel{dV})_{CD} + (c_V\,\cancel{dT} - \ell\,dV)_{DA} + \delta W = 0$

$\Rightarrow h\,dP + c_P\,dT = c_V\,dT + \ell\,dV$

C



$$\Delta U = (c_p dT + h\,dP)_{AB} + (c_p dT + h\,dP)_{BC} + (c_v dT + l\,dV)_{CD} - (c_v dT + l\,dV)_{DA} + W = 0$$

$$W \propto dP\,dV \ll \delta Q$$

$$h\,dP + c_p\,dT = c_v\,dT + l\,dV.$$

5°)

(a). $$T(P,V) \Rightarrow dT = \left.\frac{\partial T}{\partial P}\right)_V dP + \left.\frac{\partial T}{\partial V}\right)_P \cdot dV$$

$$P(T,V) \Rightarrow dP = \left.\frac{\partial P}{\partial T}\right)_V \boxed{dT} + \left.\frac{\partial P}{\partial V}\right)_T \cdot dV$$

$$\Downarrow$$

$$dP = \left.\frac{\partial P}{\partial T}\right)_V \cdot \left[\left.\frac{\partial T}{\partial P}\right)_V \cdot dP + \left.\frac{\partial T}{\partial V}\right)_P \cdot dV\right] + \left.\frac{\partial P}{\partial V}\right)_T dV \Rightarrow \left.\frac{\partial P}{\partial T}\right)_V \cdot \left.\frac{\partial T}{\partial P}\right)_V = 1 \quad (1)$$

$$-\left.\frac{\partial P}{\partial T}\right)_V \cdot \left.\frac{\partial T}{\partial V}\right)_P = \left.\frac{\partial P}{\partial V}\right)_T \quad (2)$$

(b). $$\delta Q = c_v dT + l\,dV = c_p dT + h\,dP$$

$$= c_p dT + h\left[\left.\frac{\partial P}{\partial T}\right)_V \cdot dT + \left.\frac{\partial P}{\partial V}\right)_T \cdot dV\right]$$

$$= \underbrace{\left[c_p + h\left.\frac{\partial P}{\partial T}\right)_V\right]}\,dT + \underbrace{h\left.\frac{\partial P}{\partial V}\right)_T} \cdot dV$$

$$c_v = c_p + h\left.\frac{\partial P}{\partial T}\right)_V \qquad l = h\left.\frac{\partial P}{\partial V}\right)_T$$

$$\Downarrow(1) \qquad\qquad \Downarrow(2)$$

$$h = (c_v - c_p)\left.\frac{\partial T}{\partial P}\right)_V \qquad l = h \cdot \frac{-\partial P}{\partial T} \cdot \frac{\partial T}{\partial V} = (c_v - c_p)\frac{\partial T}{\partial P} \cdot \frac{-\partial P}{\partial T} \cdot \frac{\partial T}{\partial V}$$

$$l = (c_p - c_v)\left.\frac{\partial T}{\partial V}\right)_P.$$

(c). $$\delta Q = c_v dT + l\,dV = \lambda\,dP + \nu\,dV$$

$$= \lambda\left(\frac{\partial P}{\partial T} dT + \frac{\partial P}{\partial V} \cdot dV\right) + \nu\,dV$$

$$= \underbrace{\lambda\frac{\partial P}{\partial T}}\,dT + \underbrace{\left(\lambda\frac{\partial P}{\partial V} + \nu\right)}\,dV$$

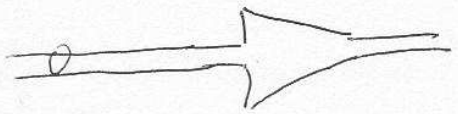
$$c_v = \lambda \cdot \frac{\partial P}{\partial T} \qquad l = \nu + \lambda \cdot \frac{\partial P}{\partial V}$$

$$\Downarrow(1) \qquad\qquad \Downarrow(2)$$

$$\lambda = c_v \cdot \left.\frac{\partial T}{\partial P}\right)_V \qquad (c_p - c_v)\left.\frac{\partial T}{\partial V}\right)_P - c_v \underbrace{\left.\frac{\partial T}{\partial P}\right)_V \cdot \frac{-\partial P}{\partial T}}_{-1} \cdot \frac{\partial T}{\partial V} = \nu$$

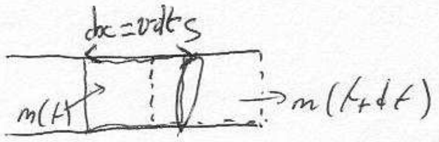
$$\nu = c_p \left.\frac{\partial T}{\partial V}\right)_P.$$

6)



CF_2Cl_2
$P_1 = 2$ bar
$T_1 = 20°C$
$\dot m = 0{,}05\ kg \cdot s^{-1}$

(a).



$$\dot m = \frac{dm}{dt} = \frac{m(t+dt) - m(t)}{dt}$$

$$m(t) = 0.$$

$$m(t+dt) = \rho V = \rho S \cdot dx \rightarrow \boxed{\dot m = \rho S \frac{dx}{dt} = \rho S \hat{v}}$$

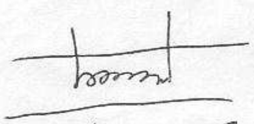
$$\text{or } \rho = \frac{PM}{RT} \rightarrow \dot m = \frac{PM}{RT} \cdot \hat{v} \cdot S.$$

(b). si $\dot m$ constant et $\hat{v} \nearrow \Rightarrow S \searrow$

or veut $\hat{v} < 10\,m s^{-1} \Rightarrow S > \dfrac{\dot m RT}{MP \cdot \hat{v}}$ et $S = \pi \dfrac{D^2}{4}$.

$$D > \left(\frac{4 \dot m RT}{\pi M P \hat{v}}\right)^{1/2} = \frac{4 \times 0{,}05 \times 8{,}314 \times (20 + 273{,}15)}{3{,}1415 \times 121 \cdot 10^{-3} \times 2 \times 10^{+5} \times 10} = \underline{\underline{2{,}53\text{ cm}}}.$$

7)



$v_e = 5\,m s^{-1}$ v_s ?
$P_e = 3{,}5$ bar $P_s = 3{,}2$ bar.
$T_e = 20°C$ $T_s = 90°C$

$$\frac{dE}{dt} = \dot Q + \dot W + \sum_{e,s} \dot m \left(h + \frac{1}{2}\hat{v}^2 + gz\right)$$

$$\text{ERP}: \quad \sum \dot m_e + \sum \dot m_s = 0 \qquad \frac{dE}{dt} = 0.$$

(a). $|\dot m_s| = |\dot m_e| \Rightarrow \rho_s \hat{v}_s S_s = \rho_e \cdot \hat{v}_e \cdot S_e \Rightarrow \hat{v}_s = \dfrac{\rho_e}{\rho_s} \cdot \hat{v}_e$

$$\text{et } \rho \propto \frac{P}{T} \left(\rho = \frac{PM}{RT}\right) \Rightarrow \boxed{\hat{v}_s = \frac{P_e}{P_s} \cdot \frac{T_s}{T_e} \hat{v}_e} = \frac{3{,}5 \times (90 + 273{,}15)}{3{,}2 \times (20 + 273{,}15)} \times 5 = \underline{\underline{6{,}77}}\ m/s$$

(b). $W = 0$
$\Delta E_p = 0$.
$\dfrac{dE}{dt} = 0$

$$0 = \dot Q + \dot m_e \left(h_e + \frac{1}{2}\hat{v}_e^2\right) + \dot m_s \left(h_s + \frac{1}{2}\hat{v}_s^2\right)$$

$$\Rightarrow \boxed{\dot Q = (h_s - h_e) + \frac{1}{2}\left(\hat{v}_s^2 - \hat{v}_e^2\right).}$$

$c_p - c_v = \dfrac{R}{M}$ $\gamma = \dfrac{c_p}{c_v}$

$\gamma - 1 = \dfrac{R}{Mc_v} \Rightarrow c_v = \dfrac{R}{(\gamma - 1)M}$

$1 - \dfrac{1}{\gamma} = \dfrac{R}{Mc_p} \Rightarrow c_p = \dfrac{\gamma}{\gamma - 1} \cdot \dfrac{R}{M}$

(c). $\Delta H = m c_p \Delta T$ ou $\Delta h = c_p \Delta T \Rightarrow h_s - h_e = c_p (T_s - T_e) = \dfrac{\gamma}{\gamma - 1} \cdot \dfrac{R}{M} (T_s - T_e)$.

$$\dot Q = \frac{\gamma}{\gamma - 1} \cdot \frac{R}{M} (T_s - T_e) + \frac{1}{2}\left(v_s^2 - v_e^2\right)$$

$$= \frac{1{,}4}{1{,}4 - 1} \cdot \frac{8{,}314}{28{,}96 \cdot 10^{-3}} (90 - 20) + \frac{1}{2}\left(6{,}77^2 - 5^2\right)$$

$$\underline{\dot Q = 70{,}35\ kJ/kg}$$

E

8) (a). $PV = nRT$ $\left.\begin{array}{l} P_0V_0 = nRT_0 \\ P_0V = nRT \end{array}\right\} \Rightarrow \frac{V}{V_0} = \frac{T}{T_0} \rightarrow$ Gay Lussac

(b). $PV = nRT$ $\left.\begin{array}{l} P_0V_0 = nRT_0 \\ PV_0 = nRT \end{array}\right\} \rightarrow \frac{P}{P_0} = \frac{T}{T_0} \rightarrow$ charles.

(c) - Gay Lussac. $\frac{V}{V_0} = \frac{T}{T_0} \Rightarrow V = V_0 \frac{T}{T_0}$.

$a = \frac{1}{V} \cdot \frac{\partial V}{\partial T}\Big)_P = \frac{T_0}{V_0 T} \cdot \frac{V_0}{T_0} = \frac{1}{T} \Rightarrow$ à P constant $\frac{1}{V} dV = \frac{1}{T} dT$

$\Rightarrow \ln V = \ln T + \ln \Upsilon(P)$

$\boxed{V = T\,\Upsilon(P)}$ $\underline{\underline{\forall P}}$

charles $\frac{P}{P_0} = \frac{T}{T_0} \Rightarrow P = P_0 \frac{T}{T_0}$.

$\beta = \frac{1}{P} \cdot \frac{\partial P}{\partial T}\Big)_V = \frac{T_0}{P_0 T} \cdot \frac{P_0}{T_0} = \frac{1}{T} \Rightarrow$ à V constant $\frac{1}{P} \cdot dP = \frac{1}{T} dT$

$\Rightarrow \ln P = \ln T + \ln \Psi(V)$.

$\boxed{P = T\,\Psi(V)}$ $\underline{\underline{\forall V}}$

donc $T = \frac{V}{\Upsilon(P)} = \frac{P}{\Psi(V)} \Leftrightarrow \underbrace{V\Psi(V)}_{\text{indép. de } P} = \underbrace{P\Upsilon(P)}_{\text{indép. de } V}$

$\Rightarrow =$ constante r

donc $\Psi(V) = \frac{r}{V}$; $\Upsilon(P) = \frac{r}{P}$.

or $P = T\,\Psi(V) = T \cdot \frac{r}{V} \Rightarrow PV = rT$
ou $V = T\,\Upsilon(P) = T \cdot \frac{r}{P} \Rightarrow PV = rT$ | Parfait.