Seismic Anisotropy: Theory, Observations and Implications

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  - Lattice-preferred orientation
- Theory
  - Elastic constant c_{ijkl} in isotropic and anisotropic media
  - Transverse isotropy
  - Azimuthal anisotropy
  - Shear wave splitting
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- Observations and Implications
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  - Anisotropy in the lithosphere and asthenosphere (LPO)
    - Anisotropy of mineral crystals
    - Anisotropy of mantle flow
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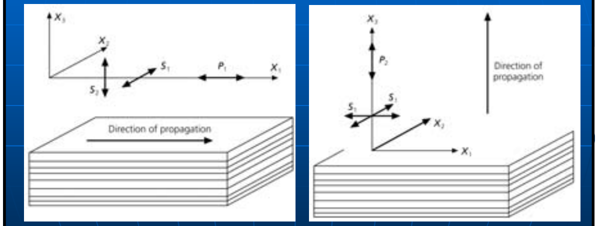
  - Anisotropy in the lower mantle
    - Mid-mantle (P660s and SKS)
    - D”
- Inner core anisotropy
  - hcp iron (ε-phase)
  - Aggregation mechanisms
    - Solidification
      - Texture
      - Magnetic field
    - Formed in the later dynamic processes
      - Convection
      - Anisotropic growth
      - Magnetic field

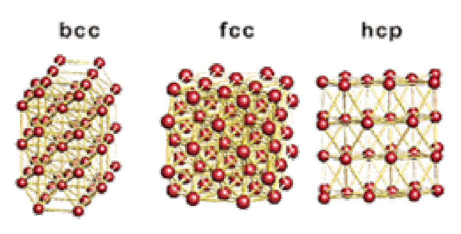
Seismic Anisotropy

- Seismic velocity function of propagation/polarization direction.
  - Anisotropic Minerals (Olivine).
  - Lattice Preferred Orientation.
  - Deformation, Flow.
- Shear wave splitting (birefringence)
  - Fast polarization direction follows flow/shear direction.
  - Delay time is a measure of layer thickness.

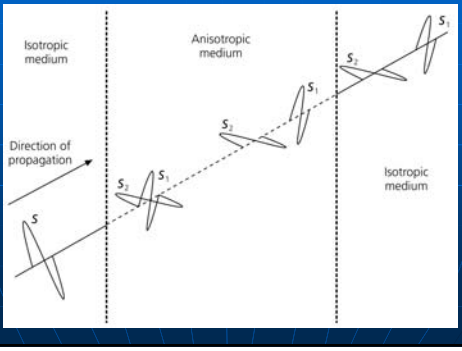
Shape-Preferred Orientation



Lattice-Preferred Orientation



Seismic Waves in an Anisotropic Medium



Theory

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl} \quad (i,j,k,l = 1,2,3)$$

$$\sigma_{ij} = \sigma_{ji}$$

$$(i,j) = \{(1,1),(2,2),(3,3),(2,3),(1,3),(1,2)\}$$

$$\varepsilon_{kl} = \varepsilon_{lk}$$

$$(k,l) = \{(1,1),(2,2),(3,3),(2,3),(1,3),(1,2)\}$$

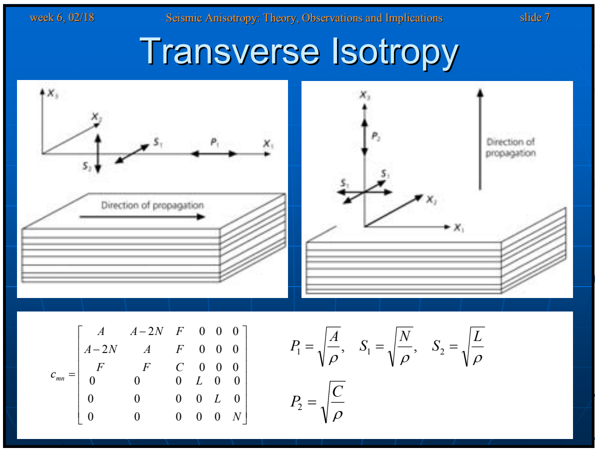
Independent elastic constants
3x3x3x3=81

6x6=36
$U=1/2(C_{ijkl}\varepsilon_{ij}\ \varepsilon_{kl})$

(36-6)/2+6=21

$$c_{mn} = \begin{bmatrix} c_{1111} & c_{1122} & c_{1133} & c_{1123} & c_{1113} & c_{1112} \\ c_{2211} & c_{2222} & c_{2233} & c_{2223} & c_{2213} & c_{2212} \\ c_{3311} & c_{3322} & c_{3333} & c_{3323} & c_{3313} & c_{3312} \\ c_{2311} & c_{2322} & c_{2333} & c_{2323} & c_{2313} & c_{2312} \\ c_{1311} & c_{1322} & c_{1333} & c_{1323} & c_{1313} & c_{1312} \\ c_{1211} & c_{1222} & c_{1233} & c_{1223} & c_{1213} & c_{1212} \end{bmatrix}$$

$$= \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} \\ c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} \end{bmatrix}$$

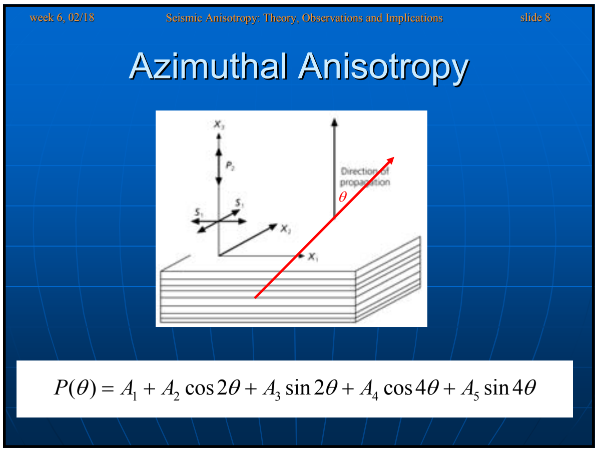


Transverse Isotropy

$$c_{mn} = \begin{bmatrix} A & A-2N & F & 0 & 0 & 0 \\ A-2N & A & F & 0 & 0 & 0 \\ F & F & C & 0 & 0 & 0 \\ 0 & 0 & 0 & L & 0 & 0 \\ 0 & 0 & 0 & 0 & L & 0 \\ 0 & 0 & 0 & 0 & 0 & N \end{bmatrix}$$

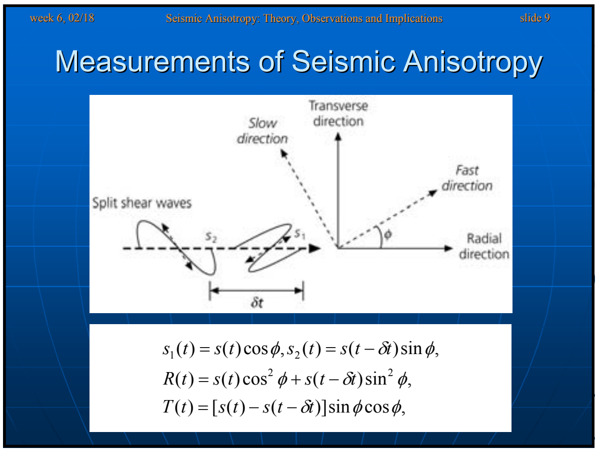
$$P_1 = \sqrt{\frac{A}{\rho}}, \quad S_1 = \sqrt{\frac{N}{\rho}}, \quad S_2 = \sqrt{\frac{L}{\rho}}$$

$$P_2 = \sqrt{\frac{C}{\rho}}$$



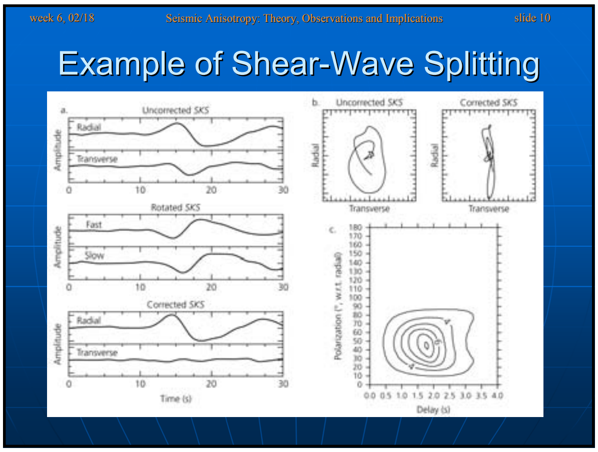
Azimuthal Anisotropy

$$P(\theta) = A_1 + A_2 \cos 2\theta + A_3 \sin 2\theta + A_4 \cos 4\theta + A_5 \sin 4\theta$$

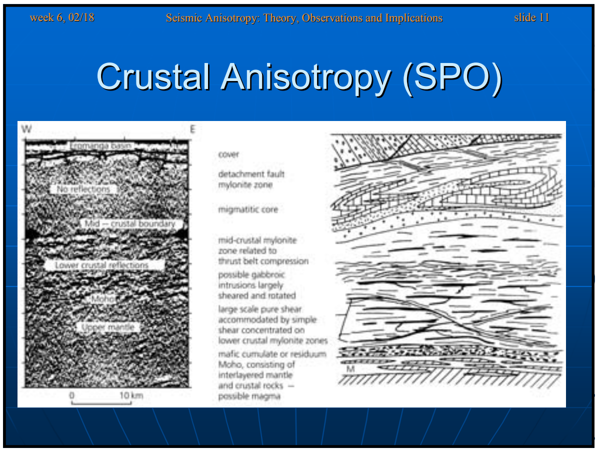


Measurements of Seismic Anisotropy

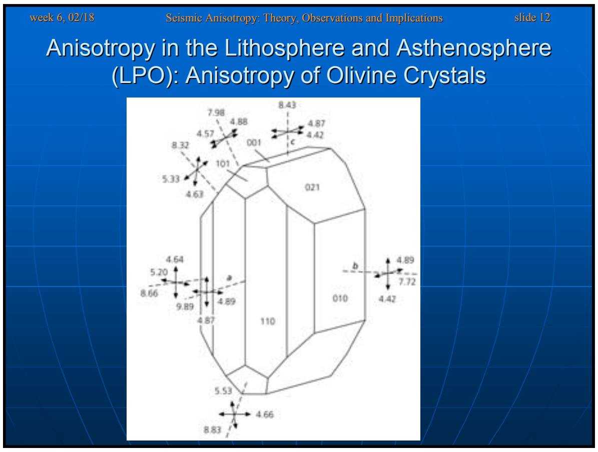
$$s_1(t) = s(t)\cos\phi, s_2(t) = s(t-\delta t)\sin\phi,$$
$$R(t) = s(t)\cos^2\phi + s(t-\delta t)\sin^2\phi,$$
$$T(t) = [s(t) - s(t-\delta t)]\sin\phi\cos\phi,$$



Example of Shear-Wave Splitting



Crustal Anisotropy (SPO)



Anisotropy in the Lithosphere and Asthenosphere (LPO): Anisotropy of Olivine Crystals

LPO and Creeps

- Dislocation creep
  - Line defects
  - High stress an/or large grain size
  - Leads to an alignment of mineral grains. The resulting aggregate is seismically anisotropic
- Diffusion creep
  - Point defects
  - Lower pressure and/or small grain size
  - Leads to a random distribution of mineral grain orientations, resulting in an effectively isotropic aggregate

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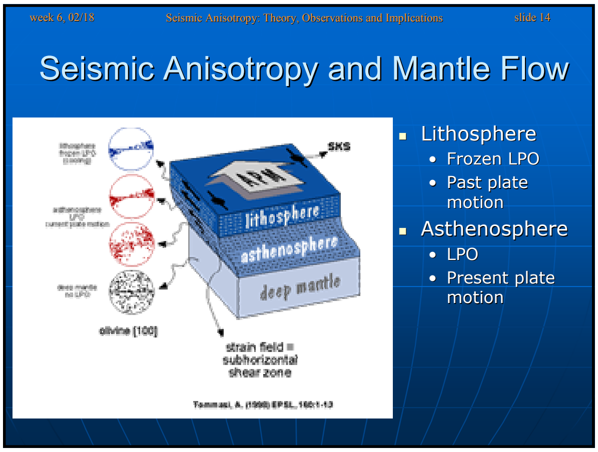
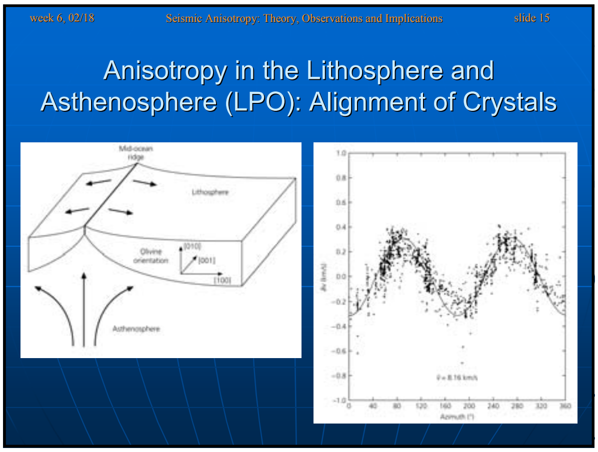
Seismic Anisotropy and Mantle Flow



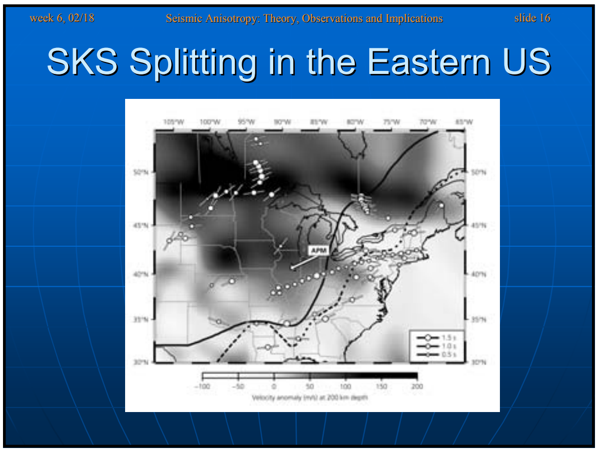
Tommasi, A. (1998) EPSL, 160:1-13

- Lithosphere
  - Frozen LPO
  - Past plate motion
- Asthenosphere
  - LPO
  - Present plate motion

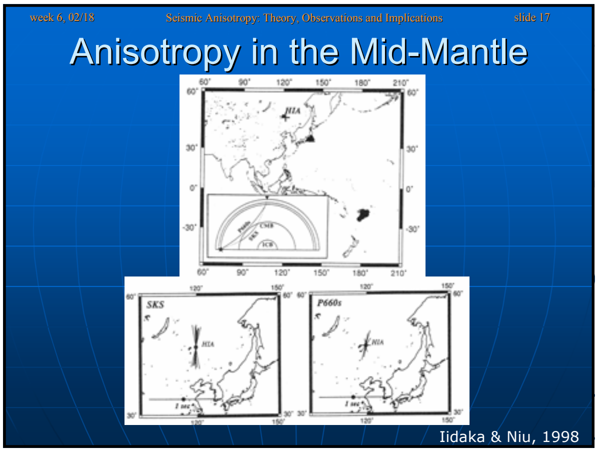
Anisotropy in the Lithosphere and Asthenosphere (LPO): Alignment of Crystals



SKS Splitting in the Eastern US

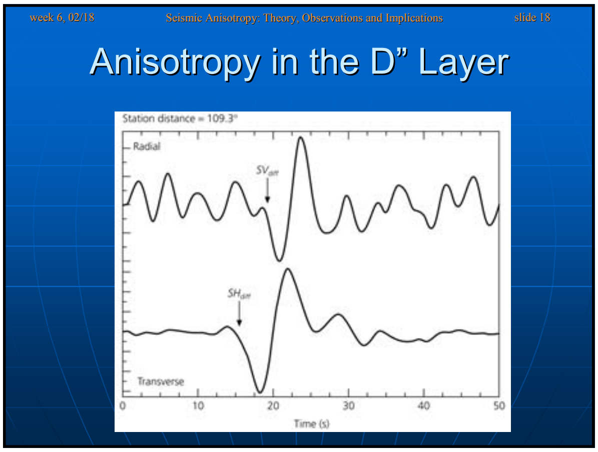


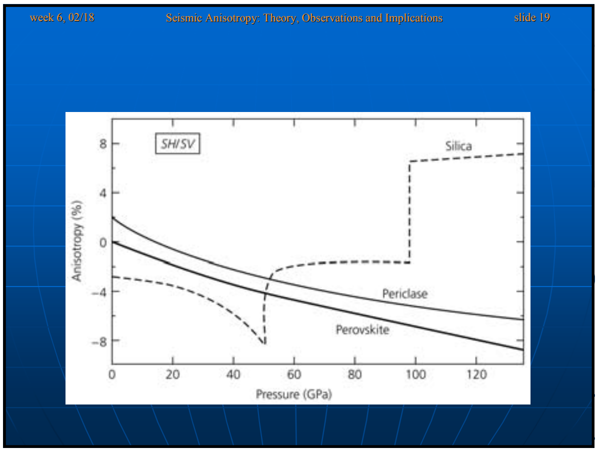
Anisotropy in the Mid-Mantle

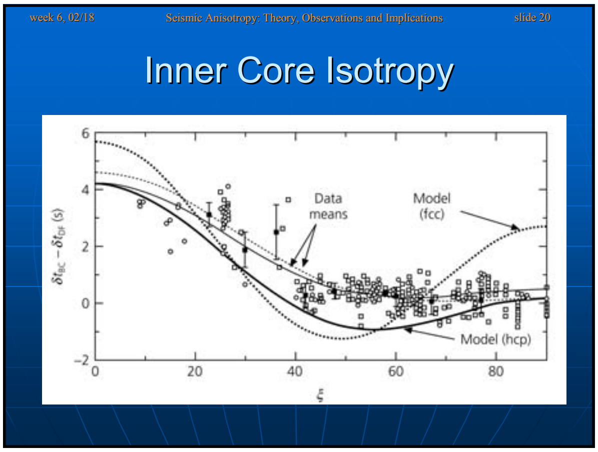


Iidaka & Niu, 1998

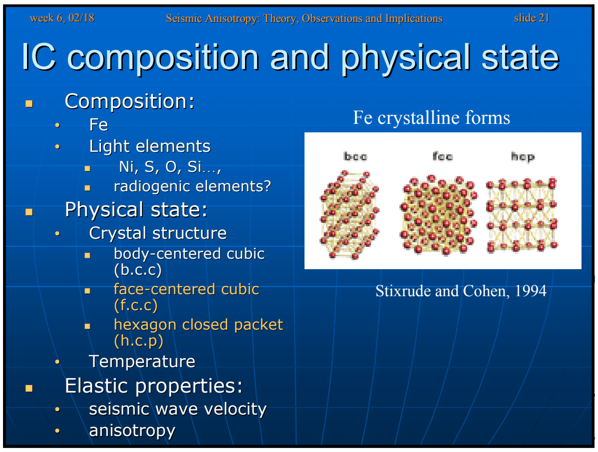
Anisotropy in the D” Layer







Inner Core Isotropy

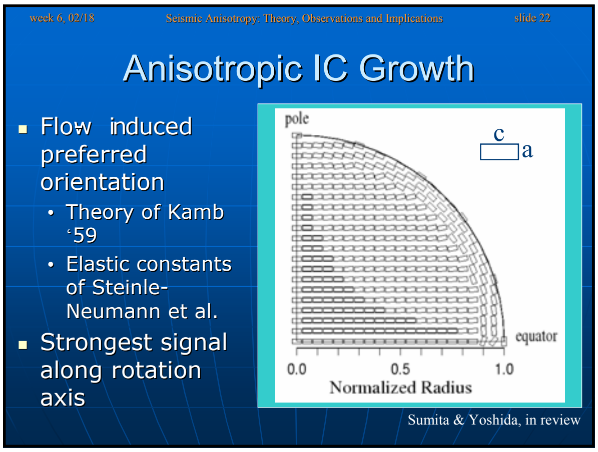


IC composition and physical state

- Composition:
  - Fe
  - Light elements
    - Ni, S, O, Si…,
    - radiogenic elements?
- Physical state:
  - Crystal structure
    - body-centered cubic (b.c.c)
    - face-centered cubic (f.c.c)
    - hexagon closed packet (h.c.p)
  - Temperature
- Elastic properties:
  - seismic wave velocity
  - anisotropy

Fe crystalline forms

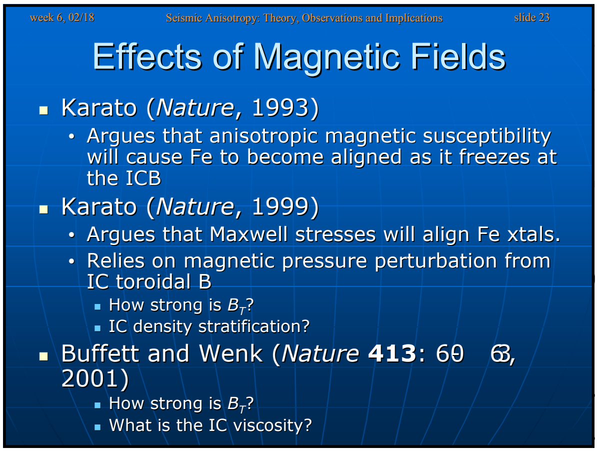
Stixrude and Cohen, 1994



Anisotropic IC Growth

- Flow induced preferred orientation
  - Theory of Kamb ‘59
  - Elastic constants of Steinle-Neumann et al.
- Strongest signal along rotation axis

Sumita & Yoshida, in review



Effects of Magnetic Fields

- Karato (*Nature*, 1993)
  - Argues that anisotropic magnetic susceptibility will cause Fe to become aligned as it freezes at the ICB
- Karato (*Nature*, 1999)
  - Argues that Maxwell stresses will align Fe xtals.
  - Relies on magnetic pressure perturbation from IC toroidal B
    - How strong is B_T?
    - IC density stratification?
- Buffett and Wenk (*Nature* **413**: 60 63, 2001)
    - How strong is B_T?
    - What is the IC viscosity?